Number systems

Number system: a set of digits (symbols) and rules to represent numbers

Positional

Number system	Radix	Symbols
Decimal	10	0, 1, 2, 3, 4, 5, 6, 7, 8, 9
Hexadecimal	16	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F
Binary	2	0, 1

Unpositional

Roman	I, V, X, L, C, D, M

The essential features of a number system are

1. Used symbols
2. Radix (b) – number of symbols
3. Representation

Decimal point

$$N_b = d_{n-1}d_{n-2}d_{n-3}\ldots d_1 d_0 . d_{-1}d_{-2}d_{-3}\ldots d_{-m}$$

4. Value

$$N_b = \sum_{i=-m}^{n-1} d_i \cdot b^i$$

Example: 1735.28 5173.82

10^3	10^2	10^1	10^0	10^{-1}	10^{-2}
1	**7**	**3**	**5**	**2**	**8**
1000	700	30	5	0.2	0.08
5000	100	70	3	0.8	0.02

1000+700+30+5+0.2+0.08=1735.28

Binary	b=2	B={0,1}
Octal	$b=8=2^3$	O={0,1,2,3,4,5,6,7}
Hexa-decimal	$b=16=2^4$	H={0,1,2,3,4,5,6,7,8, 9,A,B,C,D,E,F}

10 11 12 13 14 15

2 7 3 2

$$79_{10} = 1001111_2 = 117_8 = 4F_{16}$$

$$N_b = d_{n-1}d_{n-2}d_{n-3}\ldots d_1 d_0 . d_{-1}d_{-2}d_{-3}\ldots d_{-m}$$

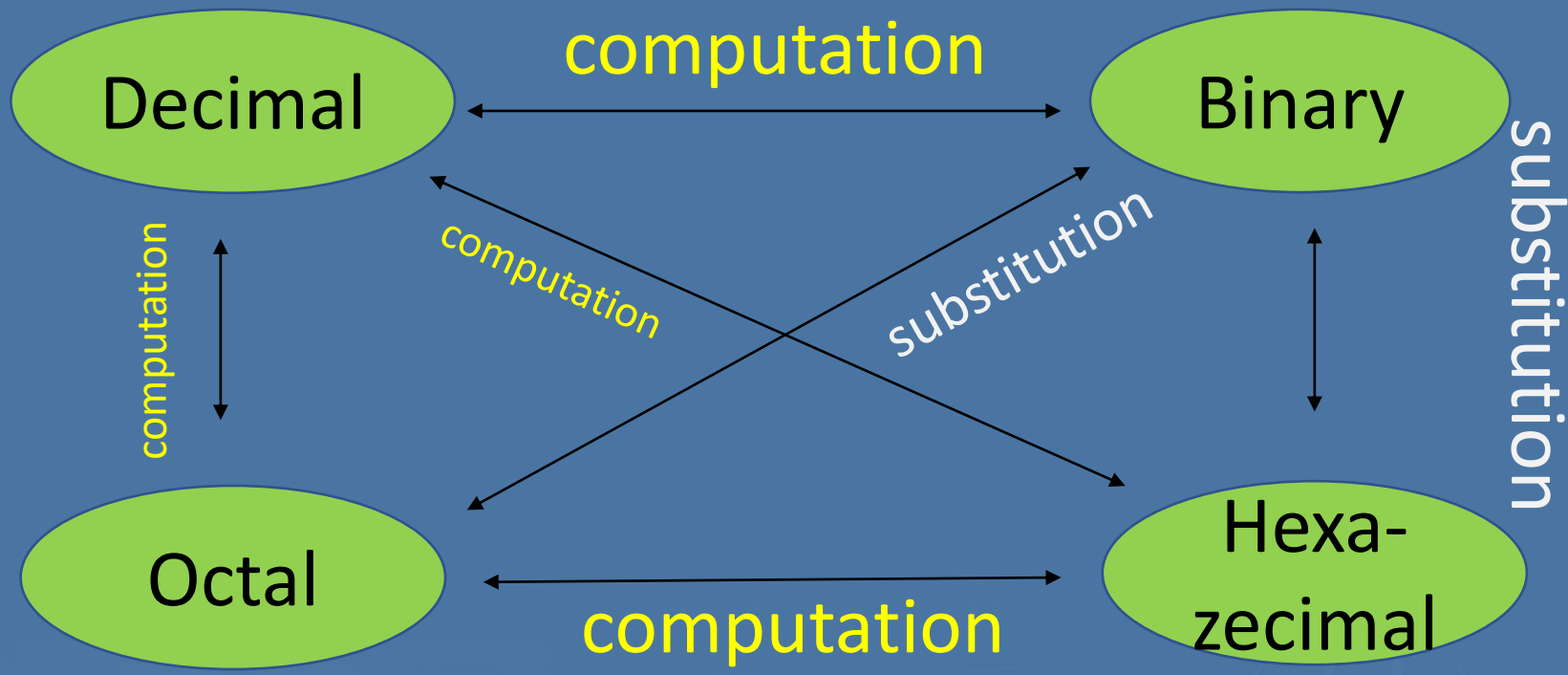
$$N_b = \sum_{i=-m}^{n-1} d_i \cdot b^i$$

$100011.101_2 = 1\cdot 2^5 + 1\cdot 2^1 + 1\cdot 2^0 + 1\cdot 2^{-1} + 1\cdot 2^{-3} = 32+2+1+ 0{,}5 + 0{,}125 = 35{,}625_{10}$

$546.2_8 = 5\cdot 8^2 + 4\cdot 8^1 + 6\cdot 8^0 + 2\cdot 8^{-1} = 320 + 32 + 6 + 0.25 = 358{,}25_{10}$

$1AE3.FC_{16} = 1\cdot 16^3 + 10\cdot 16^2 + 14\cdot 16^1 + 3\cdot 16^0 + 15\cdot 16^{-1} + 12\cdot 16^{-2} =$

Conversion


Decimal
computation
Binary
computation
computation
substitution
substitution
Octal
computation
Hexa-
zecimal

Decimal

computation

Binary

Integers

$19_{10} = 10011_2$

19:2=9 rest 1

9:2=4 rest 1

4:2=2 rest 0

2:2=1 rest 0

1:2=0 rest 1

Fractionals

$0.86_{10} = 0.01010_2$

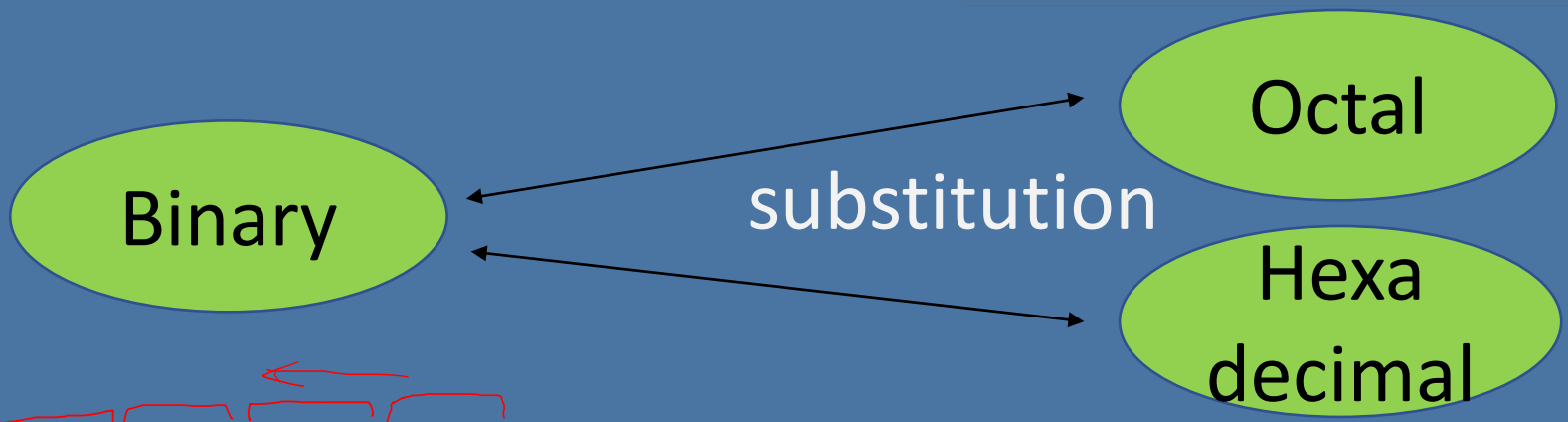
0.36*2 = 0. 72

0.72*2 = 1. 44

0.44*2 = 0. 88

0.88*2 = 1. 76

0.76*2 = 0. 52



0 11111010110 . = ?

In octal

011 111 010 110

3 7 2 6 8

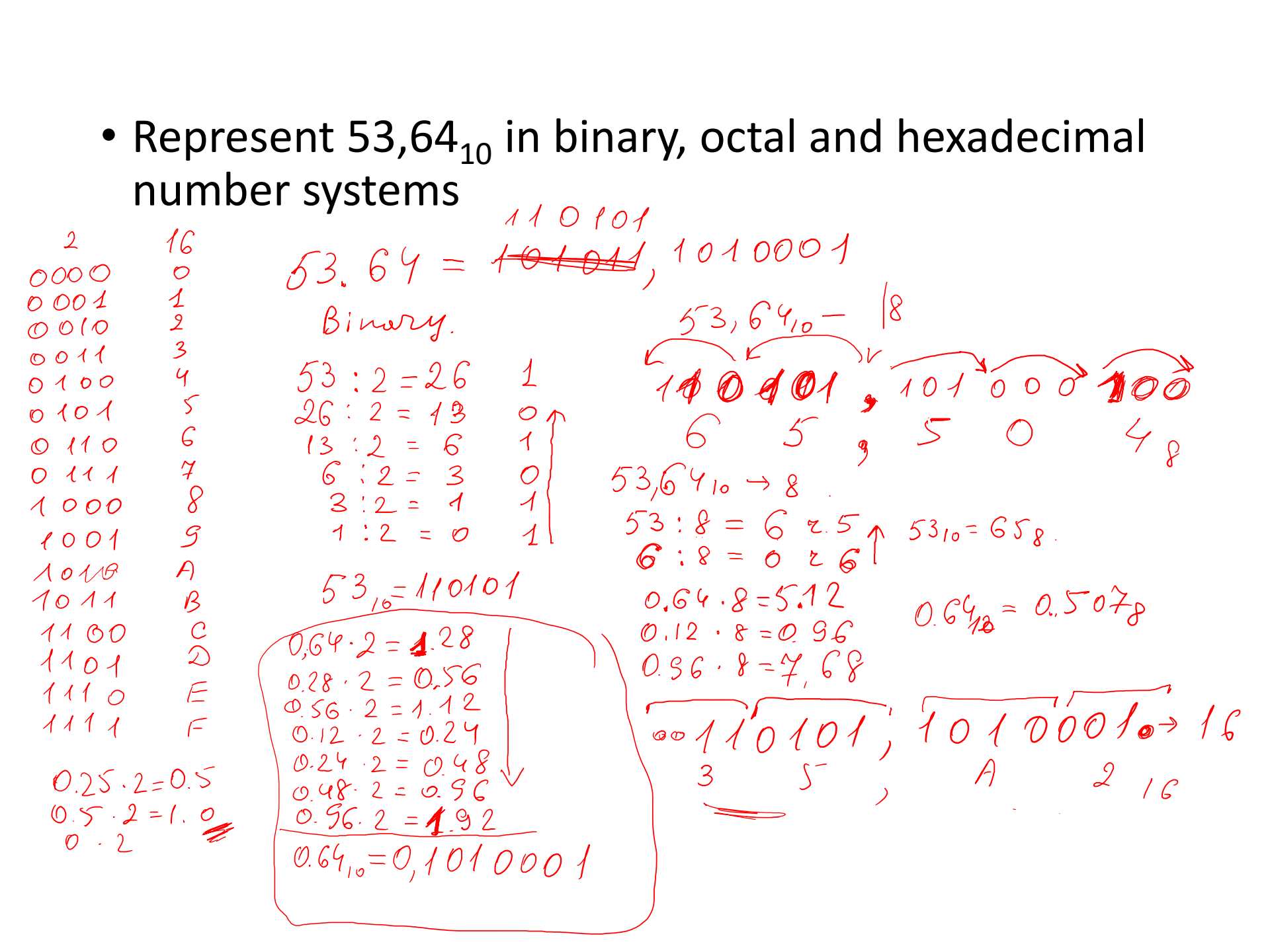
2	8
000	0
001	1
010	2
011	3
100	4
101	5
110	6
111	7

In hexadecimal

0111 1101 0110

7 D 6 16

- Represent $53{,}64_{10}$ in binary, octal and hexadecimal number systems



Convert number

$1110001101_2 = \ ?|_{16} = ?|_8 = ?|_{10}$

$3 \quad 8 \quad D_{16}$

$0011\ 1000\ 1101$

$1 \quad 6 \quad 1 \quad 5$

$9\ 8\ 7\ 6\ 5\ 4\ 3\ 2\ 1\ 0$

$1\ 1\ 1\ 0\ 0\ 0\ 1\ 1\ 0\ 1_2 = 2^9 + 2^8 + 2^7 + 2^3 + 2^2 + 2^0 =$

$512 + 256 + 128 + 8 + 4 + 1 = 909_{10}$

Convert number

$2021_{10} = ?|_{16} = ?|_{8} = ?|_{2}$

2021 2 $2021 = 11111101012 =$

3 F 5_{16}

1024	512	256	128	64	32	16	8	4	2	1	
1	1	1	1	1	1	0	1	0	1		2021

	128	64	32	16	8	4	2	1
100	0	1	1	0	0	1	0	0
255	1	1	1	1	1	1	1	1
31	0	0	0	1	1	1	1	1

$2^n - 1$

$2021_{10} = 11111101012 = 3F5_{16} =$

1 1 6 5_8

Convert number

$BEEF_{16} = ?|_8 = ?|_2 = ?|_{10}$

B E E F

$1011\ 1110\ 1110\ 1111_2$

$00\,1011\ 1110\ 1110\ 1111$

$1\ 3\ 7\ 3\ 5\ 7_8$

3 2 1 0

$BEEF = 11 \cdot 16^3 + 14 \cdot 16^2 + 14 \cdot 16^1 + 15$

Convert all terms in result number system and solve the expression

$100_{10}+100_{2}+100_{8}+100_{16}=?|_{10}$