

Number systems



Number system: a set of digits (symbols) and rules to represent numbers

Positional

Number system	Radix	Symbols
Decimal	10	0, 1, 2, 3, 4, 5, 6, 7, 8, 9
Hexadecimal	16	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F
Binary	2	0, 1

Unpositional

Roman	I, V, X, L, C, D, M
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The essential features of a number system are

1. Used symbols
2. Radix (b) – number of symbols
3. Representation

$$N_b = \underbrace{d_{n-1} d_{n-2} d_{n-3} \dots d_1 d_0}_{\text{Integer part}} . \underbrace{d_{-1} d_{-2} d_{-3} \dots d_{-m}}_{\text{Fractional part}}$$

Decimal point

4. Value

$$N_b = \sum_{i=-m}^{n-1} d_i \cdot b^i$$

Example: 1735.28 5173.82

10^3	10^2	10^1	10^0	10^{-1}	10^{-2}
1	7	3	5	2	8
1000	700	30	5	0.2	0.08
5000	100	70	3	0.8	0.02

$$1000+700+30+5+0.2+0.08=1735.28$$

Binary

$b=2$

$B=\{0,1\}$

Octal

$b=8=2^3$

$O=\{0,1,2,3,4,5,6,7\}$

Hexa-
decimal

$b=16=2^4$

$H=\{0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F\}$

10 11 12 13 14 15

$$\overset{2}{7}9_{10} = 1001111_2 = \overset{3}{1}1\overset{2}{7}_8 = 4F_{16}$$

$$N_b = \underbrace{d_{n-1}} \underbrace{d_{n-2}} \underbrace{d_{n-3}} \dots \underbrace{d_1} \underbrace{d_0} \cdot \underbrace{d_{-1}} \underbrace{d_{-2}} \underbrace{d_{-3}} \dots \underbrace{d_{-m}}$$

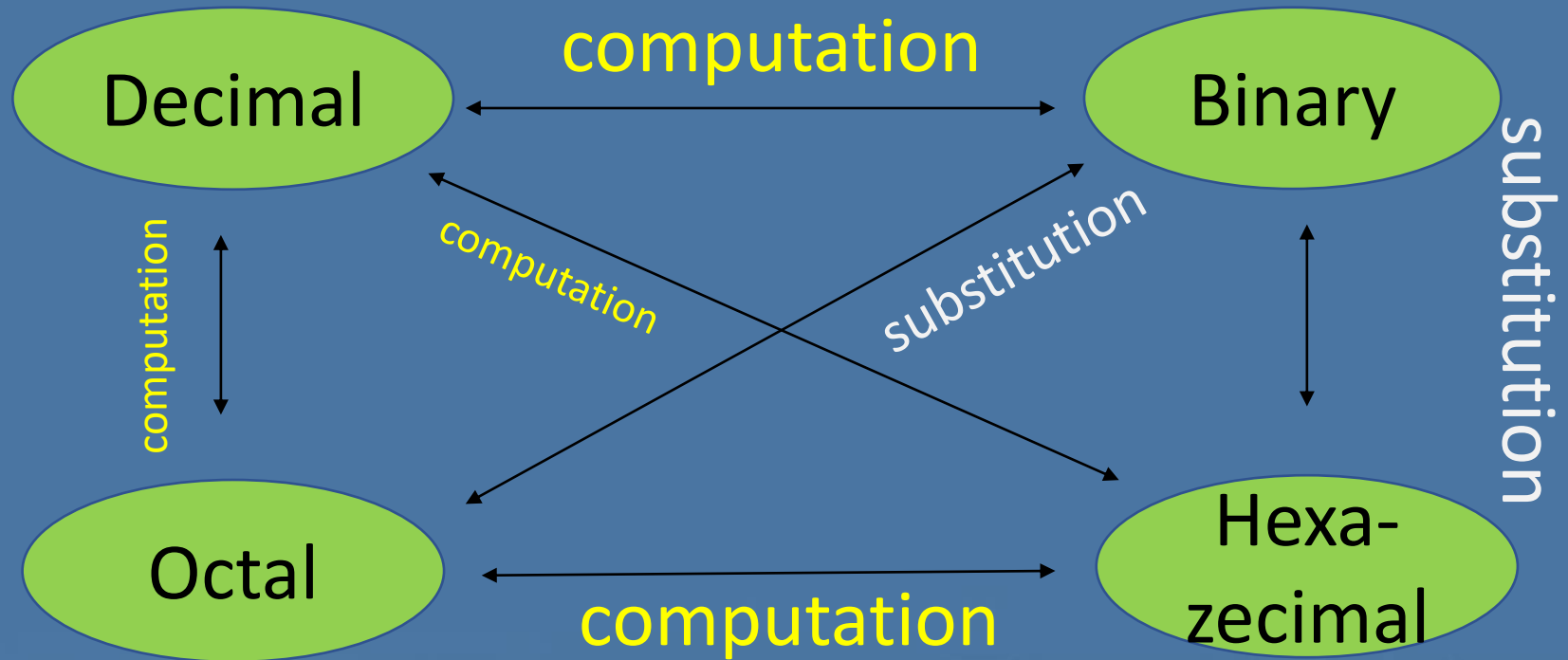
$$N_b = \sum_{i=-m}^{n-1} d_i \cdot b^i$$

$$\begin{array}{cccccccc} 5 & 4 & 3 & 2 & 1 & 0 & -1 & -2 & -3 \\ 100011.101_2 = & 1 \cdot 2^5 & + & 1 \cdot 2^4 & + & 1 \cdot 2^3 & + & 1 \cdot 2^2 & + & 1 \cdot 2^1 & + & 1 \cdot 2^0 & + & 1 \cdot 2^{-1} & + & 1 \cdot 2^{-2} & + & 1 \cdot 2^{-3} = 32 + 2 + 1 + \\ & & & & & & & & & & & & & & & & & & + 0,5 + 0,125 = 35,625_{10} \end{array}$$

$$\begin{array}{cccc} 2 & 1 & 0 & -1 \\ 546.2_8 = & 5 \cdot 8^2 & + & 4 \cdot 8^1 & + & 6 \cdot 8^0 & + & 2 \cdot 8^{-1} = 320 + 32 + 6 + 0,25 = \\ & & & & & & & & = 358,25_{10} \end{array}$$

$$\begin{array}{cccccc} 3 & 2 & 1 & 0 & -1 & -2 \\ 1AE3.FC_{16} = & 1 \cdot 16^3 & + & 10 \cdot 16^2 & + & 14 \cdot 16^1 & + & 3 \cdot 16^0 & + & 15 \cdot 16^{-1} & + & 12 \cdot 16^{-2} = \end{array}$$

Conversion



Decima
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computation

Binary

Integers

$$19_{10} = 10011_2$$

19:2=9	rest	1
9:2=4	rest	1
4:2=2	rest	0
2:2=1	rest	0
1:2=0	rest	1




Fractionals

$$0.86_{10} = 0.01010_2$$

0.86 · 2 *1 · 36*

0.36 * 2 =	0.72
0.72 * 2 =	1.44
0.44 * 2 =	0.88
0.88 * 2 =	1.76
0.76 * 2 =	0.52



Binary

Octal

Hexa decimal

substitution

011111010110. = ?

In octal

$8 = 2^3$

011 111 010 110



3

7

2

6

000	0
001	1
010	2
011	3
100	4
101	5
110	6
111	7

In hexadecimal

$16 = 2^4$

0111 1101 0110



7

D

6

16

- Represent $53,64_{10}$ in binary, octal and hexadecimal number systems

2	16
0000	0
0001	1
0010	2
0011	3
0100	4
0101	5
0110	6
0111	7
1000	8
1001	9
1010	A
1011	B
1100	C
1101	D
1110	E
1111	F

$0.25 \cdot 2 = 0.5$
$0.5 \cdot 2 = 1.0$
$0 \cdot 2 = 0$

$53,64 = \overline{10101}, 1010001$

Binary.

$53 : 2 = 26$	1
$26 : 2 = 13$	0
$13 : 2 = 6$	1
$6 : 2 = 3$	0
$3 : 2 = 1$	1
$1 : 2 = 0$	1

$53_{10} = 110101$

$0.64 \cdot 2 = 1.28$	↓
$0.28 \cdot 2 = 0.56$	
$0.56 \cdot 2 = 1.12$	
$0.12 \cdot 2 = 0.24$	
$0.24 \cdot 2 = 0.48$	
$0.48 \cdot 2 = 0.96$	
$0.96 \cdot 2 = 1.92$	
<hr/>	
$0.64_{10} = 0,1010001$	

$53,64_{10} - 18$

$\overline{110101}, 1010001$

6 5 ; 5 0 4 8

$53,64_{10} \rightarrow 8$

$53 : 8 = 6 \text{ r } 5$

$6 : 8 = 0 \text{ r } 6$

$53_{10} = 65_8$

$0.64 \cdot 8 = 5.12$

$0.12 \cdot 8 = 0.96$

$0.96 \cdot 8 = 7.68$

$0.64_{10} = 0.507_8$

$\overline{00110101}, \overline{10100010} \rightarrow 16$

3 5 ; A 2 16

Convert number

$$\underbrace{00}_{3} \underbrace{111000}_{8} \underbrace{1101}_{D}_{16} = ? |_{16} = ? |_8 = ? |_{10}$$

3 8 D₁₆

$$\begin{array}{cccccccc} 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ \hline 1 & & 6 & & 1 & & & & 5 & & & \end{array}$$

$$\begin{array}{cccccccc} 9 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{array} = 2^9 + 2^8 + 2^7 + 2^3 + 2^2 + 2^0 =$$

$$512 + 256 + 128 + 8 + 4 + 1 = 909_{10}$$

Convert number

$$2021_{10} = ?|_{16} = ?|_8 = ?|_2$$

$$2021 \quad 2 \quad 2021 = \underbrace{11111}_3 \underbrace{1010}_F \underbrace{1010}_5 \quad 2 =$$

2048	1024	512	256	128	64	32	16	8	4	2	1	
	1	1	1	1	1	1	0	1	0			2021

	128	64	32	16	8	4	2	1
100	0	1	1	0	0	1	0	0
255	1	1	1	1	1	1	1	1
31	0	0	0	1	1	1	1	1

$$2^h - 1$$

$$2021_{10} = \underbrace{11111}_3 \underbrace{1010}_F \underbrace{1010}_5 \quad 2 = 3F5_{16} =$$

Convert number

$$\text{BEEF}_{16} = ?|_8 = ?|_2 = ?|_{10}$$

B E E F

1011 1110 1110 1111₂

00 1011 1110 1110 1111 •
1 3 7 3 5 7 8

$$\begin{array}{cccc} 3 & 2 & 1 & 0 \\ \text{B} & \text{E} & \text{E} & \text{F} \end{array} = 11 \cdot 16^3 + 14 \cdot 16^2 + 14 \cdot 16^1 + 15$$

Convert all terms in result number system and solve the expression

$$100_{10} + 100_2 + 100_8 + 100_{16} = ? |_{10}$$