1a. Rezolvarea problemei cu metoda eliminarii Gauss

Fie dat sistemul de ecuatii lineare algebrice

$$Ax = b \text{ sau } \sum_{j=1}^{n} a_{ij}x_j = b_i, i = 1,2, \dots, n$$

$$w = Ax - b \text{ – vectorul rezidual}$$

Metoda eliminarii Gauss(fara pivotare)

```
A1=A; b1=b:
For k = 1:n
{
   For i = (k+1):n
    {
      mu = a(i,k)/ a(k,k);
      For j = k:n
         a(i,j) = a(i,j)-mu* a(k,j);
      b(i) = b(i)-mu* b(k);
     }
  }
     x(n)=b(n)/a(n,n);
For i = (n-1):-1:1
{
  x(i) = b(i);
  For j=(i+1):n
   {
     x(i) = x(i) - a(i,j)*x(j);
   }
  x(i) = x(i)/a(i,i);
}
For i = 1:n
{
  w(i) = 0;
  For j = 1:n
   {
     w(i) = w(i) + a1(i,j)*x(j);
   }
  w(i) = w(i) – b1(i);
}
```

1c. Rezolvarea problemei cu metoda Jacobi

$$Ax = b \iff x = Qx + c$$

$$\begin{cases} x_1 = q_{11}x_1 + q_{12}x_2 + q_{13}x_3 + \cdots + q_{1n}x_n + c_1 \\ x_2 = q_{21}x_1 + q_{22}x_2 + q_{23}x_3 + \cdots + q_{2n}x_n + c_2 \\ x_3 = q_{31}x_1 + q_{32}x_2 + q_{33}x_3 + \cdots + q_{3n}x_n + c_3 \\ \ldots \\ x_n = q_{n1}x_1 + q_{n2}x_2 + q_{n3}x_3 + \cdots + q_{nn}x_n + c_n \end{cases}$$

$$x^{(k+1)} = Qx^{(k)} + c, k = 0,1,2,3, \ldots$$

$$\begin{cases} x_1^{(k+1)} = q_{11}x_1^{(k)} + q_{12}x_2^{(k)} + q_{13}x_3^{(k)} + \cdots + q_{1n}x_n^{(k)} + c_1 \\ x_2^{(k+1)} = q_{21}x_1^{(k)} + q_{22}x_2^{(k)} + q_{23}x_3^{(k)} + \cdots + q_{2n}x_n^{(k)} + c_2 \\ x_3^{(k+1)} = q_{31}x_1^{(k)} + q_{32}x_2^{(k)} + q_{33}x_3^{(k)} + \cdots + q_{3n}x_n^{(k)} + c_3 \\ \ldots \\ x_n^{(k+1)} = q_{n1}x_1^{(k)} + q_{n2}x_2^{(k)} + q_{n3}x_3^{(k)} + \cdots + q_{nn}x_n^{(k)} + c_n \end{cases}$$

$\|Q\| = q < 1$ – conditia de convergenta

$$q = \|Q\|_\infty = \max_{i=\overline{1,n}} \sum_{j=1}^{n} |q_{ij}|$$

$$A = \begin{pmatrix} 8.7 & 0.4 & 0.6 & 0.5 \\ 0.4 & 9.2 & -0.4 & 0.8 \\ 0.6 & -0.4 & 11.4 & 1.4 \\ 0.5 & 0.8 & 1.4 & 12.6 \end{pmatrix}, b = \begin{pmatrix} 11.8 \\ 10.6 \\ 13.9 \\ -14.2 \end{pmatrix}, x^* = \begin{pmatrix} 1.2841701 \\ 1.2786261 \\ 1.3698965 \\ -1.4113366 \end{pmatrix}$$

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n = b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \cdots + a_{3n}x_n = b_3 \\ \cdots \\ a_{n1}x_1 + a_{n2}x_2 + a_{n3}x_3 + \cdots + a_{nn}x_n = b_n \end{cases}$$

$$\begin{cases} a_{11}x_1 = 0 \cdot x_1 - a_{12}x_2 - a_{13}x_3 - \cdots - a_{1n}x_n + b_1 \\ a_{22}x_2 = -a_{21}x_1 + 0 \cdot x_2 - a_{23}x_3 - \cdots - a_{2n}x_n + b_2 \\ a_{33}x_3 = -a_{31}x_1 - a_{32}x_2 + 0 \cdot x_3 - \cdots - a_{3n}x_n + b_3 \\ \cdots \\ a_{nn}x_n = -a_{n1}x_1 - a_{n2}x_2 - a_{n3}x_3 - \cdots + 0 \cdot x_n + b_n \end{cases}$$

$$\begin{cases} x_1 = 0 \cdot x_1 - \frac{a_{12}}{a_{11}}x_2 - \frac{a_{13}}{a_{11}}x_3 - \cdots - \frac{a_{1n}}{a_{11}}x_n + \frac{b_1}{a_{11}} \\ x_2 = -\frac{a_{21}}{a_{22}}x_1 + 0 \cdot x_2 - \frac{a_{23}}{a_{22}}x_3 - \cdots - \frac{a_{2n}}{a_{22}}x_n + \frac{b_2}{a_{22}} \\ x_3 = -\frac{a_{31}}{a_{33}}x_1 - \frac{a_{32}}{a_{33}}x_2 + 0 \cdot x_3 - \cdots - \frac{a_{3n}}{a_{33}}x_n + \frac{b_3}{a_{33}} \\ \cdots \\ x_n = -\frac{a_{n1}}{a_{nn}}x_1 - \frac{a_{n2}}{a_{nn}}x_2 - \frac{a_{n3}}{a_{nn}}x_3 - \cdots + 0 \cdot x_n + \frac{b_n}{a_{nn}} \end{cases}$$

$$q_{ij} = \begin{cases} -\frac{a_{ij}}{a_{ii}}, i \neq j \\ 0, i = j \end{cases}; \quad c_i = \frac{b_i}{a_{ii}}$$

$$Q = \begin{pmatrix} 0 & -0.04598 & -0.06897 & -0.05747 \\ -0.04348 & 0 & 0.04348 & -0.08696 \\ -0.05263 & 0.03509 & 0 & -0.12281 \\ -0.03968 & -0.06349 & -0.11111 & 0 \end{pmatrix}, c = \begin{pmatrix} 1.35632 \\ 1.15217 \\ 1.21930 \\ -1.12698 \end{pmatrix};$$

$$x^{(0)} = c = \begin{pmatrix} 1.35632 \\ 1.15217 \\ 1.21930 \\ -1.12698 \end{pmatrix};$$

$$k = 1: x^{(1)} = Qx^{(0)} + c = Qc + c = (Q + E)c =$$

$$= \left(\begin{pmatrix} 0 & -0.04598 & -0.06897 & -0.05747 \\ -0.04348 & 0 & 0.04348 & -0.08696 \\ -0.05263 & 0.03509 & 0 & -0.12281 \\ -0.03968 & -0.06349 & -0.11111 & 0 \end{pmatrix} + E \right) \begin{pmatrix} 1.35632 \\ 1.15217 \\ 1.21930 \\ -1.12698 \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & -0.04598 & -0.06897 & -0.05747 \\ -0.04348 & 1 & 0.04348 & -0.08696 \\ -0.05263 & 0.03509 & 1 & -0.12281 \\ -0.03968 & -0.06349 & -0.11111 & 1 \end{pmatrix} \begin{pmatrix} 1.35632 \\ 1.15217 \\ 1.21930 \\ -1.12698 \end{pmatrix} =$$

$$= \begin{pmatrix} 1.28403 \\ 1.24421 \\ 1.32674 \\ -1.38943 \end{pmatrix}$$

Criteriul de stopare ciclului: $\left\| x^* - x^{(k)} \right\| \leq \frac{q}{1-q} \left\| x^{(k)} - x^{(k-1)} \right\| < \varepsilon$

Unde $q = \|Q\|_\infty = \max_{i=\overline{1,n}} \sum_{j=1}^{n} \left| q_{ij} \right| = \max_{i=\overline{1,n}} \sum_{j=1}^{n} \left| \frac{a_{ij}}{a_{ii}} \right| = 0.21428$

$$k = 1: \frac{q}{1-q} \left\| x^{(1)} - x^{(0)} \right\|_\infty = 0.27272 * \left\| \begin{pmatrix} 1.28403 \\ 1.24421 \\ 1.32674 \\ -1.38943 \end{pmatrix} - \begin{pmatrix} 1.35632 \\ 1.15217 \\ 1.21930 \\ -1.12698 \end{pmatrix} \right\|_\infty =$$

$$= 0.27272 * \left\| \begin{pmatrix} -0.07229 \\ 0.09204 \\ 0.10744 \\ -0.26245 \end{pmatrix} \right\|_\infty = 0.27272 * 0.26245 = 0.07158 < 0.001\ ?$$

$$k = 2: x^{(2)} = Qx^{(1)} + c =$$

$$= \begin{pmatrix} 0 & 0.04598 & 0.06897 & 0.05747 \\ 0.04348 & 0 & -0.04348 & 0.08696 \\ 0.05263 & -0.03509 & 0 & 0.12281 \\ 0.03968 & 0.06349 & 0.11111 & 0 \end{pmatrix} \begin{pmatrix} 1.28403 \\ 1.24421 \\ 1.32674 \\ -1.38943 \end{pmatrix} + \begin{pmatrix} 1.35632 \\ 1.15217 \\ 1.21930 \\ -1.12698 \end{pmatrix} = \begin{pmatrix} 1.28747 \\ 1.27485 \\ 1.36601 \\ -1.40435 \end{pmatrix}$$

$$k = 2: \frac{q}{1-q} \left\| x^{(2)} - x^{(1)} \right\|_\infty = 0.27272 * \left\| \begin{pmatrix} 1.28747 \\ 1.27485 \\ 1.36601 \\ -1.40435 \end{pmatrix} - \begin{pmatrix} 1.28403 \\ 1.24421 \\ 1.32674 \\ -1.38943 \end{pmatrix} \right\|_\infty = 0.27272 * \left\| \begin{pmatrix} 0.00344 \\ 0.03064 \\ 0.03927 \\ -0.01491 \end{pmatrix} \right\|_\infty =$$

$$= 0.27272 * 0.03927 = 0.01071 < 0.001\ ?$$

Metoda Jacobi

```
criteriul=1;
While criteriul>=eps
{k++;
   For i=1:n
  { x1(i)=c(i);
     For j=1:n
 x1(i)=x1(i)+q(i,j)*x0(j);
   }
   criteriul=q/(1-q) ||x1 - x0||;
   For i=1:n ; x0(i)=x1(i);
Print “k=”; k ; “x1=”;x1(1:n); “eroarea=”; criteriul
}
```

1d. Rezolvarea problemei cu metoda Gauss-Seidel

$$Ax = b \iff x = Qx + c$$

$$A = S - T \iff (S - T)x = b \iff Sx = Tx + b \iff x = S^{-1}Tx + S^{-1}b \iff Q = S^{-1}T; c = S^{-1}b$$

$$x^{(k+1)} = Qx^{(k)} + c, k = 0,1,2,3, \dots$$

$\|Q\| = q < 1$ – conditia de convergenta

Prezentam matricea Q in forma sumei matricelor triunghiulare: $Q = L + U$

$$Q = \begin{pmatrix} q_{11} & q_{12} & \dots & q_{1n} \\ q_{21} & q_{22} & \dots & q_{2n} \\ \dots & \dots & \dots & \dots \\ q_{n1} & q_{n2} & \dots & q_{nn} \end{pmatrix} = \begin{pmatrix} 0 & 0 & \dots & 0 \\ q_{21} & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ q_{n1} & q_{n2} & \dots & 0 \end{pmatrix} + \begin{pmatrix} q_{11} & q_{12} & \dots & q_{1n} \\ 0 & q_{22} & \dots & q_{2n} \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & q_{nn} \end{pmatrix} = L + U$$

$$x^{(k+1)} = Lx^{(k+1)} + Ux^{(k)} + c, k = 0,1,2,3, \dots$$

$$\begin{cases} x_1^{(k+1)} = q_{11}x_1^{(k)} + q_{12}x_2^{(k)} + q_{13}x_3^{(k)} + \cdots + q_{1n}x_n^{(k)} + c_1 \\ x_2^{(k+1)} = q_{21}x_1^{(k+1)} + q_{22}x_2^{(k)} + q_{23}x_3^{(k)} + \cdots + q_{2n}x_n^{(k)} + c_2 \\ x_3^{(k+1)} = q_{31}x_1^{(k+1)} + q_{32}x_2^{(k+1)} + q_{33}x_3^{(k)} + \cdots + q_{3n}x_n^{(k)} + c_3 \\ \cdots \\ x_n^{(k+1)} = q_{n1}x_1^{(k+1)} + q_{n2}x_2^{(k+1)} + q_{n3}x_3^{(k+1)} + \cdots + q_{nn}x_n^{(k)} + c_n \end{cases}$$

– formule de calcul metodei Gauss-Seidel

$$\begin{cases} x_1^{(k+1)} = q_{11}x_1^{(k)} + q_{12}x_2^{(k)} + q_{13}x_3^{(k)} + \cdots + q_{1n}x_n^{(k)} + c_1 \\ x_2^{(k+1)} = q_{21}x_1^{(k)} + q_{22}x_2^{(k)} + q_{23}x_3^{(k)} + \cdots + q_{2n}x_n^{(k)} + c_2 \\ x_3^{(k+1)} = q_{31}x_1^{(k)} + q_{32}x_2^{(k)} + q_{33}x_3^{(k)} + \cdots + q_{3n}x_n^{(k)} + c_3 \\ \cdots \\ x_n^{(k+1)} = q_{n1}x_1^{(k)} + q_{n2}x_2^{(k)} + q_{n3}x_3^{(k)} + \cdots + q_{nn}x_n^{(k)} + c_n \end{cases}$$

– formule de calcul metodei Jacobi

```
criteriul=1;
While criteriul>=eps
{k++;
   For i=1:n
  { x1(i)=c(i);
     For j=1:n
     if  j>=i
       x1(i)=x1(i)+q(i,j)*x0(j);
    else
      x1(i)=x1(i)+q(i,j)*x1(j);
   }
```

Metoda Jacobi

```
criteriul=1;
While criteriul>=eps
{k++;
   For i=1:n
  { x1(i)=c(i);
     For j=1:n
       x1(i)=x1(i)+q(i,j)*x0(j);
   }
   criteriul=q/(1-q) ||x1 − x0||;
   For i=1:n ; x0(i)=x1(i);
Print "k="; k ; "x1=";x1(1:n); "eroarea="; criteriul
```

criteriul=$\frac{q}{1-q}\|x1-x0\|$***;***

For i=1:n ; x0(i)=x1(i);

Print “k=”; k ; “x1=”;x1(1:n); “eroarea=”; ***criteriul***

}

$$Q=\begin{pmatrix} 0 & -0.04598 & -0.06897 & -0.05747 \\ -0.04348 & 0 & 0.04348 & -0.08696 \\ -0.05263 & 0.03509 & 0 & -0.12281 \\ -0.03968 & -0.06349 & -0.11111 & 0 \end{pmatrix}, c=\begin{pmatrix} 1.35632 \\ 1.15217 \\ 1.21930 \\ -1.12698 \end{pmatrix};$$

$$L=\begin{pmatrix} 0 & 0 & 0 & 0 \\ -0.04348 & 0 & 0 & 0 \\ -0.05263 & 0.03509 & 0 & 0 \\ -0.03968 & -0.06349 & -0.11111 & 0 \end{pmatrix};$$

$$U=\begin{pmatrix} 0 & -0.04598 & -0.06897 & -0.05747 \\ 0 & 0 & 0.04348 & -0.08696 \\ 0 & 0 & 0 & -0.12281 \\ 0 & 0 & 0 & 0 \end{pmatrix};$$

$$x^{(0)}=c=\begin{pmatrix} 1.35632 \\ 1.15217 \\ 1.21930 \\ -1.12698 \end{pmatrix};$$

$$k=1: x^{(1)}=Lx^{(1)}+Ux^{(0)}+c$$

$$\begin{cases} x_1^{(1)} = q_{11}x_1^{(0)} + q_{12}x_2^{(0)} + q_{13}x_3^{(0)} + \cdots + q_{1n}x_n^{(0)} + c_1 \\ x_2^{(1)} = q_{21}x_1^{(1)} + q_{22}x_2^{(0)} + q_{23}x_3^{(0)} + \cdots + q_{2n}x_n^{(0)} + c_2 \\ x_3^{(1)} = q_{31}x_1^{(1)} + q_{32}x_2^{(1)} + q_{33}x_3^{(0)} + \cdots + q_{3n}x_n^{(0)} + c_3 \\ \cdots \\ x_n^{(k)} = q_{n1}x_1^{(1)} + q_{n2}x_2^{(1)} + q_{n3}x_3^{(1)} + \cdots + q_{nn}x_n^{(0)} + c_n \end{cases}$$

$$\begin{cases} x_1^{(1)} = 0 \cdot x_1^{(0)} - 0.04598x_2^{(0)} - 0.06897x_3^{(0)} - 0.05747x_4^{(0)} + 1.35632 \\ x_2^{(1)} = -0.04348x_1^{(1)} + 0 \cdot x_2^{(0)} + 0.04348x_3^{(0)} - 0.08696x_4^{(0)} + 1.15217 \\ x_3^{(1)} = -0.05263x_1^{(1)} + 0.03509x_2^{(1)} + 0 \cdot x_3^{(0)} - 0.12281x_4^{(0)} + 1.21930 \\ x_4^{(1)} = -0.03968x_1^{(1)} - 0.06349x_2^{(1)} - 0.11111x_3^{(1)} + 0 \cdot x_4^{(0)} - 1.12698 \end{cases}$$

$$x^{(0)} = c = \begin{pmatrix} 1.35632 \\ 1.15217 \\ 1.21930 \\ -1.12698 \end{pmatrix}$$

$$\begin{cases} x_1^{(1)} = 0 \cdot 1.35632 - 0.04598 \cdot 1.15217 - 0.06897 \cdot 1.21930 - 0.05747 \cdot (-1.12698) + 1.35632 = 1.28403 \\ x_2^{(1)} = -0.04348 \cdot 1.28403 + 0 \cdot 1.15217 + 0.04348 \cdot 1.21930 - 0.08696 \cdot (-1.12698) + 1.15217 = 1.24736 \\ x_3^{(1)} = -0.05263 \cdot 1.28403 + 0.03509 \cdot 1.24736 + 0 \cdot 1.21930 - 0.12281 \cdot (-1.12698) + 1.21930 = 1.33390 \\ x_4^{(1)} = -0.03968 \cdot 1.28403 - 0.06349 \cdot 1.24736 - 0.11111 \cdot 1.33390 - 0 \cdot (-1.12698) - 1.12698 = -1.40533 \end{cases}$$

$$x^{(1)}(Zeidel) = \begin{pmatrix} 1.28403 \\ 1.24736 \\ 1.33390 \\ -1.40533 \end{pmatrix}; \quad x^{(1)}(Jacobi) = \begin{pmatrix} 1.28403 \\ 1.24421 \\ 1.32674 \\ -1.38943 \end{pmatrix}; x^* = \begin{pmatrix} 1.28417 \\ 1.27863 \\ 1.36990 \\ -1.41134 \end{pmatrix}$$

$$k = 1: \frac{q}{1-q}\left\|x^{(1)} - x^{(0)}\right\|_\infty = 0.27272 * \left\|\begin{pmatrix} 1.28403 \\ 1.24736 \\ 1.33390 \\ -1.40533 \end{pmatrix} - \begin{pmatrix} 1.35632 \\ 1.15217 \\ 1.21930 \\ -1.12698 \end{pmatrix}\right\|_\infty =$$

$$= 0.27272 * \left\| \begin{pmatrix} -0.07229 \\ 0.09519 \\ 0.1460 \\ -0.27835 \end{pmatrix} \right\|_{\infty} = 0.27272 * 0.27835 = 0.075911 < 0.001 \,?$$