Metoda Cholesky

$$Ax = b$$

Să se calculeze factorizarea Cholesky pentru matricea

$$A = \begin{pmatrix} 1 & -2 & 1 \\ -2 & 8 & 4 \\ 1 & 4 & 11 \end{pmatrix}$$

Rezolvare: $A = L \cdot L^T$, $L = \begin{pmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{pmatrix}$

$$l_{11} = \sqrt{a_{11}} = 1; \quad l_{21} = \frac{a_{21}}{l_{11}} = -2; \quad l_{31} = \frac{a_{31}}{l_{11}} = 1$$

$$l_{22}^2 = a_{22} - l_{21}^2 = 8 - 4 = 4; \quad l_{22} = 2$$

$$l_{32} = \frac{a_{32} - l_{31} l_{21}}{l_{22}} = \frac{4 - 1 \cdot (-2)}{2} = \frac{6}{2} = 3$$

$$l_{33}^2 = a_{33} - l_{31}^2 - l_{32}^2 = 11 - 1 - 9 = 1;$$

$$L = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 2 & 0 \\ 1 & 3 & 1 \end{pmatrix}$$

$$A = L \cdot L^T$$

$$L \cdot L^T x = b,$$

Notam $L^T x = y$, atunci $Ly = b$

Si obtinem doua sisteme cu matrice triunghiulare.

Rezolvam primul system $Ly = b$

$$y_1 = \frac{b_1}{l_{11}}, y_i = \frac{b_i - \sum_{j=1}^{i-1} l_{ij} y_j}{l_{ii}}, i = 2,3, \dots, n$$

Rezolvam sistemul a doilea $L^T x = y$

$$x_n = \frac{y_n}{l_{nn}}, x_i = \frac{y_i - \sum_{j=i+1}^{n} l_{ji} x_j}{l_{ii}}, i = (n-1), (n-2), \dots, 2, 1$$

$$w = Ax - b \rightarrow \|w\|_\infty = \max_{i=1,2,..,n} |w_i|$$