

Индивидуальная контрольная работа по
Линейной Алгебре и Аналитической
Геометрии

Задача 1. Проверить коллинеарны или перпендикулярны векторы \vec{c}_1 и \vec{c}_2 , построенные по векторам \vec{a} и \vec{b} ?

1. $\vec{a} = \{1, -2, 3\}$, $\vec{b} = \{3, 0, -1\}$, $\vec{c}_1 = 2\vec{a} + 4\vec{b}$, $\vec{c}_2 = -\vec{a} + 3\vec{b}$.
2. $\vec{a} = \{1, 0, 1\}$, $\vec{b} = \{-2, 3, 5\}$, $\vec{c}_1 = \vec{a} + 2\vec{b}$, $\vec{c}_2 = 3\vec{a} - \vec{b}$.
3. $\vec{a} = \{-2, 4, 1\}$, $\vec{b} = \{1, -2, 7\}$, $\vec{c}_1 = 5\vec{a} + 3\vec{b}$, $\vec{c}_2 = 2\vec{a} - \vec{b}$.
4. $\vec{a} = \{1, 2, -3\}$, $\vec{b} = \{2, -1, -1\}$, $\vec{c}_1 = 4\vec{a} + 3\vec{b}$, $\vec{c}_2 = 8\vec{a} - \vec{b}$.
5. $\vec{a} = \{3, 5, 4\}$, $\vec{b} = \{5, 9, 7\}$, $\vec{c}_1 = -2\vec{a} + \vec{b}$, $\vec{c}_2 = 3\vec{a} - 2\vec{b}$.
6. $\vec{a} = \{1, 4, -2\}$, $\vec{b} = \{1, 1, -1\}$, $\vec{c}_1 = \vec{a} + \vec{b}$, $\vec{c}_2 = 4\vec{a} + 2\vec{b}$.
7. $\vec{a} = \{1, -2, 5\}$, $\vec{b} = \{3, -1, 0\}$, $\vec{c}_1 = 4\vec{a} - 2\vec{b}$, $\vec{c}_2 = -2\vec{a} + \vec{b}$.
8. $\vec{a} = \{3, 4, -1\}$, $\vec{b} = \{2, -1, 1\}$, $\vec{c}_1 = 6\vec{a} - 3\vec{b}$, $\vec{c}_2 = 2\vec{a} + 5\vec{b}$.
9. $\vec{a} = \{-2, -3, -2\}$, $\vec{b} = \{1, 0, 5\}$, $\vec{c}_1 = 3\vec{a} + 9\vec{b}$, $\vec{c}_2 = -\vec{a} - 3\vec{b}$.
10. $\vec{a} = \{-1, 4, 2\}$, $\vec{b} = \{3, -2, 6\}$, $\vec{c}_1 = 2\vec{a} - \vec{b}$, $\vec{c}_2 = 3\vec{b} - 6\vec{a}$.
11. $\vec{a} = \{5, 0, -1\}$, $\vec{b} = \{7, 2, 3\}$, $\vec{c}_1 = 2\vec{a} - \vec{b}$, $\vec{c}_2 = -6\vec{a} + 3\vec{b}$.
12. $\vec{a} = \{0, 3, -2\}$, $\vec{b} = \{1, -2, 1\}$, $\vec{c}_1 = 5\vec{a} - 2\vec{b}$, $\vec{c}_2 = 3\vec{a} + 5\vec{b}$.
13. $\vec{a} = \{-2, 7, -1\}$, $\vec{b} = \{-3, 5, 2\}$, $\vec{c}_1 = 2\vec{a} + 3\vec{b}$, $\vec{c}_2 = 3\vec{a} + 2\vec{b}$.
14. $\vec{a} = \{3, 7, 0\}$, $\vec{b} = \{1, -3, 4\}$, $\vec{c}_1 = 4\vec{a} - 2\vec{b}$, $\vec{c}_2 = \vec{b} - 2\vec{a}$.
15. $\vec{a} = \{-1, 2, -1\}$, $\vec{b} = \{2, -7, 1\}$, $\vec{c}_1 = 6\vec{a} - 2\vec{b}$, $\vec{c}_2 = \vec{b} - 3\vec{a}$.
16. $\vec{a} = \{7, 9, -2\}$, $\vec{b} = \{5, 4, 3\}$, $\vec{c}_1 = 4\vec{a} - \vec{b}$, $\vec{c}_2 = 4\vec{b} - \vec{a}$.
17. $\vec{a} = \{5, 0, -2\}$, $\vec{b} = \{6, 4, 3\}$, $\vec{c}_1 = 5\vec{a} - 3\vec{b}$, $\vec{c}_2 = 6\vec{b} - 10\vec{a}$.

$$18. \vec{a} = \{8, 3, -1\}, \vec{b} = \{4, 1, 3\}, \vec{c}_1 = 2\vec{a} - \vec{b}, \vec{c}_2 = 2\vec{b} - 4\vec{a}.$$

$$19. \vec{a} = \{3, -1, 6\}, \vec{b} = \{5, 7, 10\}, \vec{c}_1 = 4\vec{a} - 2\vec{b}, \vec{c}_2 = \vec{b} - 2\vec{a}.$$

$$20. \vec{a} = \{1, -2, 4\}, \vec{b} = \{7, 3, 5\}, \vec{c}_1 = 6\vec{a} - 3\vec{b}, \vec{c}_2 = \vec{b} - 2\vec{a}.$$

$$21. \vec{a} = \{3, 7, 0\}, \vec{b} = \{4, 6, -1\}, \vec{c}_1 = 3\vec{a} + 2\vec{b}, \vec{c}_2 = 5\vec{a} - 7\vec{b}.$$

$$22. \vec{a} = \{2, -1, 4\}, \vec{b} = \{3, -7, -6\}, \vec{c}_1 = 2\vec{a} - 3\vec{b}, \vec{c}_2 = 3\vec{a} - 2\vec{b}.$$

$$23. \vec{a} = \{5, -1, -2\}, \vec{b} = \{6, 0, 7\}, \vec{c}_1 = 3\vec{a} - 2\vec{b}, \vec{c}_2 = 4\vec{b} - 6\vec{a}.$$

$$24. \vec{a} = \{-9, 5, 3\}, \vec{b} = \{7, 1, -2\}, \vec{c}_1 = 2\vec{a} - \vec{b}, \vec{c}_2 = 3\vec{a} + 5\vec{b}.$$

$$25. \vec{a} = \{4, 2, 9\}, \vec{b} = \{0, -1, 3\}, \vec{c}_1 = 4\vec{b} - 3\vec{a}, \vec{c}_2 = 4\vec{a} - 3\vec{b}.$$

$$26. \vec{a} = \{2, -1, 6\}, \vec{b} = \{-1, 3, 8\}, \vec{c}_1 = 5\vec{a} - 2\vec{b}, \vec{c}_2 = 2\vec{a} - 5\vec{b}.$$

$$27. \vec{a} = \{5, 0, 8\}, \vec{b} = \{-3, 1, 7\}, \vec{c}_1 = 3\vec{a} - 4\vec{b}, \vec{c}_2 = 12\vec{b} - 9\vec{a}.$$

$$28. \vec{a} = \{-1, 3, 4\}, \vec{b} = \{2, -1, 0\}, \vec{c}_1 = 6\vec{a} - 2\vec{b}, \vec{c}_2 = \vec{b} - 3\vec{a}.$$

$$29. \vec{a} = \{4, 2, -7\}, \vec{b} = \{5, 0, -3\}, \vec{c}_1 = \vec{a} - 3\vec{b}, \vec{c}_2 = 6\vec{b} - 2\vec{a}.$$

$$30. \vec{a} = \{2, 0, -5\}, \vec{b} = \{1, -3, 4\}, \vec{c}_1 = 2\vec{a} - 5\vec{b}, \vec{c}_2 = 5\vec{a} - 2\vec{b}.$$

$$31. \vec{a} = \{-1, 2, -1\}, \vec{b} = \{2, -7, 1\}, \vec{c}_1 = 6\vec{a} - 2\vec{b}, \vec{c}_2 = \vec{b} - 3\vec{a}.$$

$$32. \vec{a} = \{5, 4, 1\}, \vec{b} = \{-3, 5, 2\}, \vec{c}_1 = 2\vec{a} - 4\vec{b}, \vec{c}_2 = 2\vec{b} - 3\vec{a}.$$

$$33. \vec{a} = \{-5, 0, -1\}, \vec{b} = \{2, -3, 4\}, \vec{c}_1 = 3\vec{a} + \vec{b}, \vec{c}_2 = \vec{b} - 5\vec{a}.$$

$$34. \vec{a} = \{-7, -2, -5\}, \vec{b} = \{1, -3, 0\}, \vec{c}_1 = 4\vec{a} - \vec{b}, \vec{c}_2 = 2\vec{b} - 7\vec{a}.$$

$$35. \vec{a} = \{-2, -3, -5\}, \vec{b} = \{-1, 0, 4\}, \vec{c}_1 = \vec{a} + 5\vec{b}, \vec{c}_2 = 12\vec{b} - 3\vec{a}.$$

$$36. \vec{a} = \{-1, 4, 3\}, \vec{b} = \{3, -3, 4\}, \vec{c}_1 = 5\vec{a} - 2\vec{b}, \vec{c}_2 = 7\vec{b} - 9\vec{a}.$$

$$37. \vec{a} = \{5, 7, -2\}, \vec{b} = \{1, -3, 3\}, \vec{c}_1 = 3\vec{a} + 2\vec{b}, \vec{c}_2 = 2\vec{b} - 9\vec{a}.$$

$$38. \vec{a} = \{1, -4, 6\}, \vec{b} = \{3, -3, \}, \vec{c}_1 = 2\vec{a} - 3\vec{b}, \vec{c}_2 = 3\vec{b} - 8\vec{a}.$$

$$39. \vec{a} = \{1, -3, 1\}, \vec{b} = \{-2, -4, 4\}, \vec{c}_1 = 7\vec{a} - 4\vec{b}, \vec{c}_2 = 8\vec{b} - 3\vec{a}.$$

$$40. \vec{a} = \{4, 5, -1\}, \vec{b} = \{1, -3, 1\}, \vec{c}_1 = \vec{a} - 2\vec{b}, \vec{c}_2 = 12\vec{b} + 5\vec{a}.$$

$$41. \vec{a} = \{7, 2, 1\}, \vec{b} = \{3, -3, 5\}, \vec{c}_1 = 2\vec{a} + 3\vec{b}, \vec{c}_2 = 5\vec{b} + 6\vec{a}.$$

$$42. \vec{a} = \{-3, 0, -1\}, \vec{b} = \{4, -3, 7\}, \vec{c}_1 = 3\vec{a} + \vec{b}, \vec{c}_2 = 10\vec{b} - 3\vec{a}.$$

$$43. \vec{a} = \{2, -5, 1\}, \vec{b} = \{-7, -3, 4\}, \vec{c}_1 = -2\vec{a} + 6\vec{b}, \vec{c}_2 = \vec{b} - 3\vec{a}.$$

$$44. \vec{a} = \{4, -5, -3\}, \vec{b} = \{-3, 2, 4\}, \vec{c}_1 = 8\vec{a} - 3\vec{b}, \vec{c}_2 = 5\vec{b} - 6\vec{a}.$$

$$45. \vec{a} = \{-4, 3, -5\}, \vec{b} = \{2, 7, -4\}, \vec{c}_1 = 9\vec{a} - 4\vec{b}, \vec{c}_2 = 4\vec{b} + \vec{a}.$$

$$46. \vec{a} = \{-5, -2, 1\}, \vec{b} = \{4, -3, 2\}, \vec{c}_1 = 2\vec{a} - 3\vec{b}, \vec{c}_2 = -5\vec{b} + 4\vec{a}.$$

$$47. \vec{a} = \{-2, -1, -3\}, \vec{b} = \{5, -6, 3\}, \vec{c}_1 = -3\vec{a} - \vec{b}, \vec{c}_2 = 9\vec{b} + 3\vec{a}.$$

$$48. \vec{a} = \{5, -6, 3\}, \vec{b} = \{-6, -1, 3\}, \vec{c}_1 = -\vec{a} + 7\vec{b}, \vec{c}_2 = -\vec{b} + 5\vec{a}.$$

$$49. \vec{a} = \{8, -3, -2\}, \vec{b} = \{-5, 1, 2\}, \vec{c}_1 = 6\vec{a} - \vec{b}, \vec{c}_2 = 4\vec{b} - 7\vec{a}.$$

$$50. \vec{a} = \{-1, 6, -7\}, \vec{b} = \{3, 8, -5\}, \vec{c}_1 = 8\vec{a} - 7\vec{b}, \vec{c}_2 = 2\vec{b} - 7\vec{a}.$$

Задача 2.

Вычислить:

- 1) Скалярное произведение (\vec{a}, \vec{b}) ;
- 2) Найти косинус угла между векторами \vec{a} и \vec{b} ;
- 3) Вычислить площадь параллелограмма, построенного на векторах \vec{a} и \vec{b} .

$$1. \vec{a} = \vec{p} + 2\vec{q}, \vec{b} = 3\vec{p} - \vec{q}, |\vec{p}| = 1, |\vec{q}| = 2, \left(\overset{\wedge}{\vec{p}, \vec{q}} \right) = \pi/6.$$

$$2. \vec{a} = 3\vec{p} + \vec{q}, \vec{b} = \vec{p} - 2\vec{q}, |\vec{p}| = 4, |\vec{q}| = 1, \left(\overset{\wedge}{\vec{p}, \vec{q}} \right) = \pi/4.$$

$$3. \vec{a} = \vec{p} - 3\vec{q}, \vec{b} = \vec{p} + 2\vec{q}, |\vec{p}| = 1/5, |\vec{q}| = 1, \left(\overset{\wedge}{\vec{p}, \vec{q}} \right) = \pi/2.$$

$$4. \vec{a} = 3\vec{p} - 2\vec{q}, \vec{b} = \vec{p} + 5\vec{q}, |\vec{p}| = 4, |\vec{q}| = 1/2, \left(\vec{p}, \vec{q}\right) = 5\pi/6.$$

$$5. \vec{a} = \vec{p} - 2\vec{q}, \vec{b} = 2\vec{p} + \vec{q}, |\vec{p}| = 2, |\vec{q}| = 3, \left(\vec{p}, \vec{q}\right) = 3\pi/4.$$

$$6. \vec{a} = \vec{p} + 3\vec{q}, \vec{b} = \vec{p} - 2\vec{q}, |\vec{p}| = 2, |\vec{q}| = 3, \left(\vec{p}, \vec{q}\right) = \pi/3.$$

$$7. \vec{a} = 2\vec{p} - \vec{q}, \vec{b} = \vec{p} + 3\vec{q}, |\vec{p}| = 3, |\vec{q}| = 2, \left(\vec{p}, \vec{q}\right) = \pi/2.$$

$$8. \vec{a} = 4\vec{p} + \vec{q}, \vec{b} = \vec{p} - \vec{q}, |\vec{p}| = 7, |\vec{q}| = 2, \left(\vec{p}, \vec{q}\right) = \pi/4.$$

$$9. \vec{a} = \vec{p} - 4\vec{q}, \vec{b} = 3\vec{p} + \vec{q}, |\vec{p}| = 1, |\vec{q}| = 2, \left(\vec{p}, \vec{q}\right) = \pi/6.$$

$$10. \vec{a} = \vec{p} + 4\vec{q}, \vec{b} = 2\vec{p} - \vec{q}, |\vec{p}| = 7, |\vec{q}| = 2, \left(\vec{p}, \vec{q}\right) = \pi/3.$$

$$11. \vec{a} = 3\vec{p} + 2\vec{q}, \vec{b} = \vec{p} - \vec{q}, |\vec{p}| = 10, |\vec{q}| = 1, \left(\vec{p}, \vec{q}\right) = \pi/2.$$

$$12. \vec{a} = 4\vec{p} - \vec{q}, \vec{b} = \vec{p} + 2\vec{q}, |\vec{p}| = 5, |\vec{q}| = 4, \left(\vec{p}, \vec{q}\right) = \pi/4.$$

$$13. \vec{a} = 2\vec{p} + 3\vec{q}, \vec{b} = \vec{p} - 2\vec{q}, |\vec{p}| = 6, |\vec{q}| = 7, \left(\vec{p}, \vec{q}\right) = \pi/3.$$

$$14. \vec{a} = 3\vec{p} - \vec{q}, \vec{b} = \vec{p} + 2\vec{q}, |\vec{p}| = 3, |\vec{q}| = 4, \left(\vec{p}, \vec{q}\right) = \pi/3.$$

$$15. \vec{a} = 2\vec{p} + 3\vec{q}, \vec{b} = \vec{p} - 2\vec{q}, |\vec{p}| = 2, |\vec{q}| = 3, \left(\vec{p}, \vec{q}\right) = \pi/4.$$

$$16. \vec{a} = 2\vec{p} - 3\vec{q}, \vec{b} = 3\vec{p} + \vec{q}, |\vec{p}| = 4, |\vec{q}| = 1, \left(\vec{p}, \vec{q}\right) = \pi/6.$$

$$17. \vec{a} = 5\vec{p} + \vec{q}, \vec{b} = \vec{p} - 3\vec{q}, |\vec{p}| = 1, |\vec{q}| = 2, \left(\vec{p}, \vec{q}\right) = \pi/3.$$

$$18. \vec{a} = 7\vec{p} - 2\vec{q}, \vec{b} = \vec{p} + 3\vec{q}, |\vec{p}| = 1/2, |\vec{q}| = 2, (\vec{p}, \vec{q}) = \pi/2.$$

$$19. \vec{a} = 6\vec{p} - \vec{q}, \vec{b} = \vec{p} + \vec{q}, |\vec{p}| = 3, |\vec{q}| = 4, (\vec{p}, \vec{q}) = \pi/4.$$

$$20. \vec{a} = 10\vec{p} + \vec{q}, \vec{b} = 3\vec{p} - 2\vec{q}, |\vec{p}| = 4, |\vec{q}| = 1, (\vec{p}, \vec{q}) = \pi/6.$$

$$21. \vec{a} = 6\vec{p} - \vec{q}, \vec{b} = \vec{p} + 2\vec{q}, |\vec{p}| = 8, |\vec{q}| = 1/2, (\vec{p}, \vec{q}) = \pi/3.$$

$$22. \vec{a} = 3\vec{p} + 4\vec{q}, \vec{b} = \vec{q} - \vec{p}, |\vec{p}| = 5/2, |\vec{q}| = 2, (\vec{p}, \vec{q}) = \pi/2.$$

$$23. \vec{a} = 7\vec{p} + \vec{q}, \vec{b} = \vec{p} - 3\vec{q}, |\vec{p}| = 3, |\vec{q}| = 1, (\vec{p}, \vec{q}) = 3\pi/4.$$

$$24. \vec{a} = \vec{p} + 3\vec{q}, \vec{b} = 3\vec{p} - \vec{q}, |\vec{p}| = 3, |\vec{q}| = 5, (\vec{p}, \vec{q}) = 2\pi/3.$$

$$25. \vec{a} = 3\vec{p} + \vec{q}, \vec{b} = \vec{p} - 3\vec{q}, |\vec{p}| = 7, |\vec{q}| = 2, (\vec{p}, \vec{q}) = \pi/4.$$

$$26. \vec{a} = 5\vec{p} - \vec{q}, \vec{b} = \vec{p} + \vec{q}, |\vec{p}| = 5, |\vec{q}| = 3, (\vec{p}, \vec{q}) = 5\pi/6.$$

$$27. \vec{a} = 3\vec{p} - 4\vec{q}, \vec{b} = \vec{p} + 3\vec{q}, |\vec{p}| = 2, |\vec{q}| = 3, (\vec{p}, \vec{q}) = \pi/4.$$

$$28. \vec{a} = 6\vec{p} - \vec{q}, \vec{b} = \vec{p} + 5\vec{q}, |\vec{p}| = 1/2, |\vec{q}| = 4, (\vec{p}, \vec{q}) = 5\pi/6.$$

$$29. \vec{a} = 2\vec{p} + 3\vec{q}, \vec{b} = \vec{p} - 2\vec{q}, |\vec{p}| = 2, |\vec{q}| = 1, (\vec{p}, \vec{q}) = \pi/3.$$

$$30. \vec{a} = 2\vec{p} - 3\vec{q}, \vec{b} = 5\vec{p} + \vec{q}, |\vec{p}| = 2, |\vec{q}| = 3, (\vec{p}, \vec{q}) = \pi/2.$$

$$31. \vec{a} = -5\vec{p} - 4\vec{q}, \vec{b} = 3\vec{q} + 6\vec{p}, |\vec{p}| = 3, |\vec{q}| = 5, (\vec{p}, \vec{q}) = 5\pi/6.$$

$$32. \vec{a} = -2\vec{p} + 3\vec{q}, \vec{b} = 4\vec{q} - \vec{p}, |\vec{p}| = 1, |\vec{q}| = 3, (\vec{p}, \vec{q}) = \pi/6.$$

- 33.** $\vec{a} = 5\vec{p} - 2\vec{q}$, $b = -3\vec{q} - \vec{p}$, $|\vec{p}| = 4$, $|\vec{q}| = 5$, $\left(\vec{p}, \vec{q}\right) = 3\pi/4$
- 34.** $\vec{a} = 5\vec{p} + 2\vec{q}$, $b = -6\vec{q} - 4\vec{p}$, $|\vec{p}| = 3$, $|\vec{q}| = 2$, $\left(\vec{p}, \vec{q}\right) = 5\pi/6$.
- 35.** $\vec{a} = 3\vec{p} - 2\vec{q}$, $b = -4\vec{q} + 5\vec{p}$, $|\vec{p}| = 2$, $|\vec{q}| = 3$, $\left(\vec{p}, \vec{q}\right) = \pi/3$.
- 36.** $\vec{a} = 2\vec{p} - 5\vec{q}$, $b = -3\vec{q} + 4\vec{p}$, $|\vec{p}| = 2$, $|\vec{q}| = 4$, $\left(\vec{p}, \vec{q}\right) = 2\pi/3$.
- 37.** $\vec{a} = 3\vec{p} + 2\vec{q}$, $b = -4\vec{q} - 6\vec{p}$, $|\vec{p}| = 2$, $|\vec{q}| = 5$, $\left(\vec{p}, \vec{q}\right) = 2\pi/3$.
- 38.** $\vec{a} = 5\vec{p} + 2\vec{q}$, $b = \vec{q} - 4\vec{p}$, $|\vec{p}| = -4$, $|\vec{q}| = 3$, $\left(\vec{p}, \vec{q}\right) = \pi$.
- 39.** $\vec{a} = -3\vec{p} - 2\vec{q}$, $b = \vec{q} + 5\vec{p}$, $|\vec{p}| = 3$, $|\vec{q}| = 6$, $\left(\vec{p}, \vec{q}\right) = 5\pi/6$.
- 40.** $\vec{a} = 5\vec{p} - 3\vec{q}$, $b = 4\vec{q} + 2\vec{p}$, $|\vec{p}| = 4$, $|\vec{q}| = 1$, $\left(\vec{p}, \vec{q}\right) = 2\pi/3$.
- 41.** $\vec{a} = -2\vec{p} + 3\vec{q}$, $b = 3\vec{q} - 6\vec{p}$, $|\vec{p}| = 6$, $|\vec{q}| = 3$, $\left(\vec{p}, \vec{q}\right) = 5\pi/6$.
- 42.** $\vec{a} = -2\vec{p} - 4\vec{q}$, $b = 3\vec{q} + \vec{p}$, $|\vec{p}| = 3$, $|\vec{q}| = 2$, $\left(\vec{p}, \vec{q}\right) = 7\pi/12$.
- 43.** $\vec{a} = 4\vec{p} + 3\vec{q}$, $b = -\vec{q} + 2\vec{p}$, $|\vec{p}| = 4$, $|\vec{q}| = 5$, $\left(\vec{p}, \vec{q}\right) = 3\pi/4$.
- 44.** $\vec{a} = -2\vec{p} + 3\vec{q}$, $b = 5\vec{q} + \vec{p}$, $|\vec{p}| = 2$, $|\vec{q}| = 5$, $\left(\vec{p}, \vec{q}\right) = 2\pi/3$.
- 45.** $\vec{a} = 4\vec{p} - 3\vec{q}$, $\vec{b} = 5\vec{q} + 2\vec{p}$, $|\vec{p}| = 3$, $|\vec{q}| = 5$, $\left(\vec{p}, \vec{q}\right) = 2\pi/3$.
- 46.** $\vec{a} = -3\vec{p} + 2\vec{q}$, $b = 4\vec{q} - 7\vec{p}$, $|\vec{p}| = 2$, $|\vec{q}| = 5$, $\left(\vec{p}, \vec{q}\right) = 5\pi/6$.
- 47.** $\vec{a} = -3\vec{p} + 5\vec{q}$, $b = -3\vec{q} + 6\vec{p}$, $|\vec{p}| = 4$, $|\vec{q}| = 1$, $\left(\vec{p}, \vec{q}\right) = 7\pi/12$.

48. $\vec{a} = 5\vec{p} - 2\vec{q}$, $b = -2\vec{q} + 3\vec{p}$, $|\vec{p}| = 7$, $|\vec{q}| = 3$, $\left(\vec{p}, \vec{q}\right) = 3\pi/4$.

49. $\vec{a} = 2\vec{p} - 3\vec{q}$, $b = -5\vec{q} + 6\vec{p}$, $|\vec{p}| = 6$, $|\vec{q}| = 3$, $\left(\vec{p}, \vec{q}\right) = \pi/3$.

50. $\vec{a} = 5\vec{p} - 8\vec{q}$, $\vec{b} = 4\vec{q} - 5\vec{p}$, $|\vec{p}| = 12$, $|\vec{q}| = 2$, $\left(\vec{p}, \vec{q}\right) = 2\pi/3$.

Задача 3. Доказать, что векторы \vec{p} , \vec{q} , \vec{r} образуют базис и написать разложение вектора \vec{x} по базису $\{\vec{p}, \vec{q}, \vec{r}\}$.

1. $\vec{x} = \{-2, 4, 7\}$, $\vec{p} = \{0, 1, 2\}$, $\vec{q} = \{1, 0, 1\}$, $\vec{r} = \{-1, 2, 4\}$.

2. $\vec{x} = \{6, 12, -1\}$, $\vec{p} = \{1, 3, 0\}$, $\vec{q} = \{2, -1, 1\}$, $\vec{r} = \{0, -1, 2\}$.

3. $\vec{x} = \{1, -4, 4\}$, $\vec{p} = \{2, 1, -1\}$, $\vec{q} = \{0, 3, 2\}$, $\vec{r} = \{1, -1, 1\}$.

4. $\vec{x} = \{-9, 5, 5\}$, $\vec{p} = \{4, 1, 1\}$, $\vec{q} = \{2, 0, -3\}$, $\vec{r} = \{-1, 2, 1\}$.

5. $\vec{x} = \{-5, -5, 5\}$, $\vec{p} = \{-2, 0, 1\}$, $\vec{q} = \{1, 3, -1\}$, $\vec{r} = \{0, 4, 1\}$.

6. $\vec{x} = \{13, 2, 7\}$, $\vec{p} = \{5, 1, 0\}$, $\vec{q} = \{2, -1, 3\}$, $\vec{r} = \{1, 0, -1\}$.

7. $\vec{x} = \{-19, -1, 7\}$, $\vec{p} = \{0, 1, 1\}$, $\vec{q} = \{-2, 0, 1\}$, $\vec{r} = \{3, 1, 0\}$.

8. $\vec{x} = \{3, -3, 4\}$, $\vec{p} = \{1, 0, 2\}$, $\vec{q} = \{0, 1, 1\}$, $\vec{r} = \{2, -1, 4\}$.

9. $\vec{x} = \{3, 3, -1\}$, $\vec{p} = \{3, 1, 0\}$, $\vec{q} = \{-1, 2, 1\}$, $\vec{r} = \{-1, 0, 2\}$.

10. $\vec{x} = \{-1, 7, -4\}$, $\vec{p} = \{-1, 2, 1\}$, $\vec{q} = \{2, 0, 3\}$, $\vec{r} = \{1, 1, -1\}$.

11. $\vec{x} = \{6, 5, -14\}$, $\vec{p} = \{1, 1, 4\}$, $\vec{q} = \{0, -3, 2\}$, $\vec{r} = \{2, 1, -1\}$.

12. $\vec{x} = \{6, -1, 7\}$, $\vec{p} = \{1, -2, 0\}$, $\vec{q} = \{-1, 1, 3\}$, $\vec{r} = \{1, 0, 4\}$.

13. $\vec{x} = \{5, 15, 0\}$, $\vec{p} = \{1, 0, 5\}$, $\vec{q} = \{-1, 3, 2\}$, $\vec{r} = \{0, -1, 1\}$.

14. $\vec{x} = \{2, -1, 11\}$, $\vec{p} = \{1, 0, 1\}$, $\vec{q} = \{0, 1, -2\}$, $\vec{r} = \{1, 0, 3\}$.

$$15. \vec{x} = \{11, 5, -3\}, \vec{p} = \{1, 0, 2\}, \vec{q} = \{-1, 0, 1\}, \vec{r} = \{2, 5, -3\}.$$

$$16. \vec{x} = \{8, 0, 5\}, \vec{p} = \{2, 0, 1\}, \vec{q} = \{1, 1, 0\}, \vec{r} = \{4, 1, 2\}.$$

$$17. \vec{x} = \{3, 1, 8\}, \vec{p} = \{0, 1, 3\}, \vec{q} = \{1, 2, -1\}, \vec{r} = \{2, 0, -1\}.$$

$$18. \vec{x} = \{8, 1, 12\}, \vec{p} = \{1, 2, -1\}, \vec{q} = \{3, 0, 2\}, \vec{r} = \{-1, 1, 1\}.$$

$$19. \vec{x} = \{-9, -8, -3\}, \vec{p} = \{1, 4, 1\}, \vec{q} = \{-3, 2, 0\}, \vec{r} = \{1, -1, 2\}.$$

$$20. \vec{x} = \{-5, 9, -13\}, \vec{p} = \{0, 1, -2\}, \vec{q} = \{3, -1, 1\}, \vec{r} = \{4, 1, 0\}.$$

$$21. \vec{x} = \{-15, 5, 6\}, \vec{p} = \{0, 5, 1\}, \vec{q} = \{3, 2, -1\}, \vec{r} = \{-1, 1, 0\}.$$

$$22. \vec{x} = \{8, 9, 4\}, \vec{p} = \{1, 0, 1\}, \vec{q} = \{0, -2, 1\}, \vec{r} = \{1, 3, 0\}.$$

$$23. \vec{x} = \{23, -14, -30\}, \vec{p} = \{2, 1, 0\}, \vec{q} = \{1, -1, 0\}, \vec{r} = \{-3, 2, 5\}.$$

$$24. \vec{x} = \{3, 1, 3\}, \vec{p} = \{2, 1, 0\}, \vec{q} = \{1, 0, 1\}, \vec{r} = \{4, 2, 1\}.$$

$$25. \vec{x} = \{-1, 7, 0\}, \vec{p} = \{0, 3, 1\}, \vec{q} = \{3, 0, 1\}, \vec{r} = \{2, -1, 0\}.$$

$$26. \vec{x} = \{11, -1, 4\}, \vec{p} = \{1, -1, 2\}, \vec{q} = \{3, 2, 0\}, \vec{r} = \{-1, 1, 1\}.$$

$$27. \vec{x} = \{-13, 2, 18\}, \vec{p} = \{1, 1, 4\}, \vec{q} = \{-3, 0, 2\}, \vec{r} = \{1, 2, -1\}.$$

$$28. \vec{x} = \{0, -8, 9\}, \vec{p} = \{0, -2, 1\}, \vec{q} = \{3, 1, -1\}, \vec{r} = \{4, 0, 1\}.$$

$$29. \vec{x} = \{8, -7, -13\}, \vec{p} = \{0, 1, 5\}, \vec{q} = \{3, -1, 2\}, \vec{r} = \{-1, 0, 1\}.$$

$$30. \vec{x} = \{2, 7, 5\}, \vec{p} = \{1, 0, 1\}, \vec{q} = \{1, -2, 0\}, \vec{r} = \{0, 3, 1\}.$$

$$31. \vec{x} = \{-13, 2, 18\}, \vec{p} = \{1, 1, 4\}, \vec{q} = \{-3, 0, 2\}, \vec{r} = \{1, 2, -1\}.$$

$$32. \vec{x} = \{7, 23, 4\}, \vec{p} = \{5, 4, 1\}, \vec{q} = \{-3, 5, 2\}, \vec{r} = \{2, -1, 3\}.$$

$$33. \vec{x} = \{0, 11, -14\}, \vec{p} = \{2, -1, 4\}, \vec{q} = \{-3, 0, -2\}, \vec{r} = \{4, 5, -3\}.$$

$$34. \vec{x} = \{28, -19, -7\}, \vec{p} = \{-1, 1, 2\}, \vec{q} = \{2, -3, -5\}, \vec{r} = \{-6, 3, -1\}.$$

$$35. \vec{x} = \{16, 6, 15\}, \vec{p} = \{-7, -2, -4\}, \vec{q} = \{-4, 0, 3\}, \vec{r} = \{3, 1, 2\}.$$

$$36. \vec{x} = \{15, -15, 24\}, \vec{p} = \{5, 1, 2\}, \vec{q} = \{-2, 1, -3\}, \vec{r} = \{4, -3, 5\}.$$

$$37. \vec{x} = \{-19, -5, -4\}, \vec{p} = \{0, 2, -3\}, \vec{q} = \{4, -3, -2\}, \vec{r} = \{-5, -4, 0\}.$$

- 38.** $\vec{x} = \{-3, 2, -3\}$, $\vec{p} = \{3, -1, 2\}$, $\vec{q} = \{-2, 3, 1\}$, $\vec{r} = \{4, -5, -3\}$.
- 39.** $\vec{x} = \{-9, 34, -20\}$, $\vec{p} = \{5, 3, 1\}$, $\vec{q} = \{-1, 2, -3\}$, $\vec{r} = \{3, -4, 2\}$.
- 40.** $\vec{x} = \{1, 12, -20\}$, $\vec{p} = \{3, 1, -3\}$, $\vec{q} = \{-2, 4, 1\}$, $\vec{r} = \{1, -2, 5\}$.
- 41.** $\vec{x} = \{15, 6, -17\}$, $\vec{p} = \{6, 1, -3\}$, $\vec{q} = \{-3, 2, 1\}$, $\vec{r} = \{-1, -3, 4\}$.
- 42** $\vec{x} = \{-12, 14, -31\}$, $\vec{p} = \{4, 2, 3\}$, $\vec{q} = \{-3, 1, -8\}$, $\vec{r} = \{2, -4, 5\}$.
- 43.** $\vec{x} = \{-2, 17, 5\}$, $\vec{p} = \{1, 3, 6\}$, $\vec{q} = \{-3, 4, -5\}$, $\vec{r} = \{1, -7, 2\}$.
- 44.** $\vec{x} = \{-5, 11, -15\}$, $\vec{p} = \{11, 1, 2\}$, $\vec{q} = \{-3, 3, 4\}$, $\vec{r} = \{-4, -2, 7\}$.
- 45.** $\vec{x} = \{-10, -13, 8\}$, $\vec{p} = \{9, 5, 3\}$, $\vec{q} = \{-3, 2, 1\}$, $\vec{r} = \{4, -7, 4\}$.
- 46.** $\vec{x} = \{1, 7, -12\}$, $\vec{p} = \{5, 4, -3\}$, $\vec{q} = \{-3, 2, -5\}$, $\vec{r} = \{-1, 3, -4\}$.
- 47.** $\vec{x} = \{6, 9, -1\}$, $\vec{p} = \{8, 3, -5\}$, $\vec{q} = \{-6, 7, -3\}$, $\vec{r} = \{4, -1, 7\}$.
- 48.** $\vec{x} = \{3, -4, 6\}$, $\vec{p} = \{11, -3, 4\}$, $\vec{q} = \{-13, 2, -6\}$, $\vec{r} = \{5, -3, 8\}$.
- 49.** $\vec{x} = \{-6, 10, 23\}$, $\vec{p} = \{1, 11, 12\}$, $\vec{q} = \{-2, 2, 3\}$, $\vec{r} = \{-5, -3, 8\}$.
- 50.** $\vec{x} = \{-5, 6, 7\}$, $\vec{p} = \{6, 4, 8\}$, $\vec{q} = \{-13, 5, -6\}$, $\vec{r} = \{2, -3, 5\}$.

Задача 4. Даны силы

$$\vec{F}_1 = \{x_1, y_1, z_1\}, \vec{F}_2 = \{x_2, y_2, z_2\}, \vec{F}_3 = \{x_3, y_3, z_3\},$$

приложенные в точке $A(x_A, y_A, z_A)$, и точка

$B(x_B, y_B, z_B)$. Вычислить:

- 1) Работу равнодействующих $\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$ этих сил, когда её точка приложения $A(x_A, y_A, z_A)$, двигаясь прямолинейно, перемещается в точку $B(x_B, y_B, z_B)$;
- 2) Модуль и направляющие косинусы момента равнодействующей этих сил относительно точки $B(x_B, y_B, z_B)$.

$$1. \vec{F}_1 = \{-2, 4, 7\}, \vec{F}_2 = \{0, 1, 2\}, \vec{F}_3 = \{1, 0, 1\}, A(-1, 3, 7), B(5, -2, 1).$$

$$2. \vec{F}_1 = \{6, -7, 5\}, \vec{F}_2 = \{-5, 1, 3\}, \vec{F}_3 = \{2, 1, -7\}, A(-2, 4, 5), B(6, -5, -1).$$

- 3.** $\vec{F}_1 = \{-1, 1, 5\}$, $\vec{F}_2 = \{0, 1, -2\}$, $\vec{F}_3 = \{8, -2, 1\}$, A(-1,3,7), B(4,-2,1).
- 4.** $\vec{F}_1 = \{2, 3, -1\}$, $\vec{F}_2 = \{0, 1, -2\}$, $\vec{F}_3 = \{1, 0, 1\}$, A(-3,3,7), B(-5,2,8).
- 5.** $\vec{F}_1 = \{-2, 4, 1\}$, $\vec{F}_2 = \{0, -1, 4\}$, $\vec{F}_3 = \{1, 0, -1\}$, A(1,-3,6), B(4,-4,1).
- 6.** $\vec{F}_1 = \{-2, 1, 5\}$, $\vec{F}_2 = \{0, 1, -2\}$, $\vec{F}_3 = \{1, 2, 3\}$, A(-5,3,7), B(1,-2,1).
- 7.** $\vec{F}_1 = \{2, -5, 8\}$, $\vec{F}_2 = \{0, 1, -3\}$, $\vec{F}_3 = \{1, 0, -2\}$, A(-1,3,7), B(5,-2,1).
- 8.** $\vec{F}_1 = \{-7, 4, 3\}$, $\vec{F}_2 = \{0, 1, -6\}$, $\vec{F}_3 = \{-1, 0, 1\}$, A(-8,3,2), B(6,-3,1).
- 9.** $\vec{F}_1 = \{-7, 2, 1\}$, $\vec{F}_2 = \{0, -1, 2\}$, $\vec{F}_3 = \{5, 0, 3\}$, A(1,-3,7), B(4,2,-1).
- 10.** $\vec{F}_1 = \{2, -4, 5\}$, $\vec{F}_2 = \{0, -1, 2\}$, $\vec{F}_3 = \{1, 5, 1\}$, A(-1,3,7), B(5,-1,1).
- 11.** $\vec{F}_1 = \{-1, 3, 5\}$, $\vec{F}_2 = \{0, -1, 2\}$, $\vec{F}_3 = \{5, 0, 1\}$, A(-3,5,1), B(4,-6,1).
- 12.** $\vec{F}_1 = \{4, -8, 7\}$, $\vec{F}_2 = \{0, -2, 2\}$, $\vec{F}_3 = \{-1, 0, 2\}$, A(-3,3,7), B(8,-2,1).
- 13.** $\vec{F}_1 = \{-2, 1, 7\}$, $\vec{F}_2 = \{0, -1, 2\}$, $\vec{F}_3 = \{1, -1, 3\}$, A(1,-3,2), B(8,-1,1).
- 14.** $\vec{F}_1 = \{-4, 4, 1\}$, $\vec{F}_2 = \{0, -1, 2\}$, $\vec{F}_3 = \{-2, 0, 4\}$, A(-1,2,7), B(3,-2,1).
- 15.** $\vec{F}_1 = \{2, -1, 6\}$, $\vec{F}_2 = \{0, -2, 2\}$, $\vec{F}_3 = \{1, -1, 1\}$, A(2,1,7), B(4,2,-1).
- 16.** $\vec{F}_1 = \{-1, 4, 7\}$, $\vec{F}_2 = \{0, 2, -2\}$, $\vec{F}_3 = \{4, 1, 1\}$, A(8,-3,7), B(8,-2,1).
- 17.** $\vec{F}_1 = \{-6, 4, 7\}$, $\vec{F}_2 = \{1, 0, 2\}$, $\vec{F}_3 = \{-1, 3, 2\}$, A(1,3,5), B(7,-1,1).
- 18.** $\vec{F}_1 = \{-4, 4, 8\}$, $\vec{F}_2 = \{0, -4, 2\}$, $\vec{F}_3 = \{-6, 5, 1\}$, A(1,3,-3), B(1,-2,1).
- 19.** $\vec{F}_1 = \{2, -5, 6\}$, $\vec{F}_2 = \{0, 1, 2\}$, $\vec{F}_3 = \{5, 2, -1\}$, A(6,3,-1), B(3,-2,1).
- 20.** $\vec{F}_1 = \{-2, 4, 7\}$, $\vec{F}_2 = \{0, 1, -3\}$, $\vec{F}_3 = \{1, 2, 1\}$, A(5,-3,7), B(-5,3,4).
- 21.** $\vec{F}_1 = \{2, -4, 1\}$, $\vec{F}_2 = \{0, -1, 2\}$, $\vec{F}_3 = \{1, 5, -3\}$, A(1,3,-1), B(1,-2,1).
- 22.** $\vec{F}_1 = \{-1, 6, 7\}$, $\vec{F}_2 = \{0, -3, 2\}$, $\vec{F}_3 = \{1, 0, -1\}$, A(1,2,-8), B(-4,-2,6).
- 23.** $\vec{F}_1 = \{2, -2, 7\}$, $\vec{F}_2 = \{0, 1, 2\}$, $\vec{F}_3 = \{2, 8, -1\}$, A(1,-5,7), B(5,-5,1).
- 24.** $\vec{F}_1 = \{3, 1, -1\}$, $\vec{F}_2 = \{0, -1, 1\}$, $\vec{F}_3 = \{5, 2, -1\}$, A(4,0,5), B(4,1,6).
- 25.** $\vec{F}_1 = \{6, 2, -1\}$, $\vec{F}_2 = \{0, -2, 1\}$, $\vec{F}_3 = \{5, 3, -1\}$, A(4,4,5), B(2,1,6).

- 26.** $\vec{F}_1 = \{5, -2, 1\}$, $\vec{F}_2 = \{0, -2, 1\}$, $\vec{F}_3 = \{1, 0, -8\}$, A(4,-4,5), B(2,1,2).
- 27.** $\vec{F}_1 = \{1, -3, 2\}$, $\vec{F}_2 = \{0, 1, 1\}$, $\vec{F}_3 = \{9, 5, -1\}$, A(1,2,-6), B(-3,1,2).
- 28.** $\vec{F}_1 = \{1, -3, 5\}$, $\vec{F}_2 = \{0, -1, 1\}$, $\vec{F}_3 = \{1, 2, 1\}$, A(1,3,-4), B(-4,2,2).
- 29.** $\vec{F}_1 = \{2, 7, -7\}$, $\vec{F}_2 = \{0, 4, 2\}$, $\vec{F}_3 = \{1, 0, -1\}$, A(1,-2,5), B(3,4,-1).
- 30.** $\vec{F}_1 = \{2, 7, -8\}$, $\vec{F}_2 = \{0, -4, 2\}$, $\vec{F}_3 = \{1, -2, 1\}$, A(1,-6,5), B(3,-2,1).
- 31.** $\vec{F}_1 = \{3, -5, 8\}$, $\vec{F}_2 = \{0, 4, 2\}$, $\vec{F}_3 = \{1, 2, -3\}$, A(1,-5,7), B(2,-2,1).
- 32.** $\vec{F}_1 = \{2, 4, -5\}$, $\vec{F}_2 = \{0, 1, -6\}$, $\vec{F}_3 = \{2, -1, 1\}$, A(-1,3,7), B(5,-2,1).
- 33.** $\vec{F}_1 = \{-1, 4, 9\}$, $\vec{F}_2 = \{2, 1, 1\}$, $\vec{F}_3 = \{1, 2, -1\}$, A(-3,3,5), B(5,2,-1).
- 34.** $\vec{F}_1 = \{-1, 4, 5\}$, $\vec{F}_2 = \{0, 1, -1\}$, $\vec{F}_3 = \{1, 0, -1\}$, A(-1,3,5), B(5,1,-1).
- 35.** $\vec{F}_1 = \{4, -2, 7\}$, $\vec{F}_2 = \{0, 1, -2\}$, $\vec{F}_3 = \{1, 0, 1\}$, A(-1,3,8), B(5,2,-1).
- 36.** $\vec{F}_1 = \{-4, 4, 6\}$, $\vec{F}_2 = \{0, 1, -2\}$, $\vec{F}_3 = \{1, 0, -1\}$, A(1,3,-7), B(4,-2,2).
- 37.** $\vec{F}_1 = \{2, -4, 7\}$, $\vec{F}_2 = \{0, 1, 3\}$, $\vec{F}_3 = \{1, 0, 2\}$, A(1,-3,7), B(5,-5,1).
- 38.** $\vec{F}_1 = \{1, -5, 4\}$, $\vec{F}_2 = \{1, -3, -2\}$, $\vec{F}_3 = \{5, 4, -3\}$, A(1,2,-9), B(6,2,-3).
- 39.** $\vec{F}_1 = \{3, 5, -4\}$, $\vec{F}_2 = \{1, -6, -2\}$, $\vec{F}_3 = \{5, 8, -3\}$, A(1,5,-9), B(6,3,-3).
- 40.** $\vec{F}_1 = \{1, 8, 6\}$, $\vec{F}_2 = \{-5, 4, 1\}$, $\vec{F}_3 = \{-9, -1, 6\}$, A(4,6,8), B(-1,-2,6).
- 41.** $\vec{F}_1 = \{3, 4, -5\}$, $\vec{F}_2 = \{8, 3, -9\}$, $\vec{F}_3 = \{-2, 5, 8\}$, A(5,3,-1), B(11,0,-3).
- 42.** $\vec{F}_1 = \{6, -2, 5\}$, $\vec{F}_2 = \{7, -2, 10\}$, $\vec{F}_3 = \{-5, 7, 6\}$, A(7,-2,5), B(8,2,-5).
- 43.** $\vec{F}_1 = \{3, -1, 8\}$, $\vec{F}_2 = \{-6, 2, 11\}$, $\vec{F}_3 = \{7, -4, 8\}$, A(-1,3,6), B(9,-3,4).
- 44.** $\vec{F}_1 = \{-2, 7, 3\}$, $\vec{F}_2 = \{5, -1, 6\}$, $\vec{F}_3 = \{-4, 1, -5\}$, A(7,2,-4), B(8,2,-7).
- 45.** $\vec{F}_1 = \{5, -7, 9\}$, $\vec{F}_2 = \{-2, 1, 8\}$, $\vec{F}_3 = \{-8, -5, 2\}$, A(5,1,-6), B(-1,3,4).
- 46.** $\vec{F}_1 = \{2, -3, 5\}$, $\vec{F}_2 = \{1, 8, -3\}$, $\vec{F}_3 = \{0, 6, -3\}$, A(1,-5,9), B(-1,5,-4).
- 47.** $\vec{F}_1 = \{-1, 9, 2\}$, $\vec{F}_2 = \{4, -6, 1\}$, $\vec{F}_3 = \{7, 5, -1\}$, A(8,-3,-4), B(-3,-6,2).
- 48.** $\vec{F}_1 = \{3, 0, 2\}$, $\vec{F}_2 = \{-4, 1, -2\}$, $\vec{F}_3 = \{11, -3, 5\}$, A(6,-2,5), B(-2,5,9).

49. $\vec{F}_1 = \{4, 1, -8\}$, $\vec{F}_2 = \{4, 3, -5\}$, $\vec{F}_3 = \{2, 1, 5\}$, A(-1,2,-3), B(-7,3,-2).

50. $\vec{F}_1 = \{-3, 1, 8\}$, $\vec{F}_2 = \{1, -2, 2\}$, $\vec{F}_3 = \{3, -2, -5\}$, A(-3,3,6), B(8,-3,4).

Задача 5. Даны вершины $\triangle ABC$:

$A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$. Найти:

- 1) уравнение стороны AB;
- 2) уравнение высоты CH;
- 3) уравнение медианы AM;
- 4) координаты точки N пересечения медианы AM и высоты CH;
- 5) уравнение прямой, проходящей через вершину C, параллельно стороне AB;
- 6) расстояние от точки C до прямой AB.

1. A(-2, 4), B(3, 1), C(10, 7);

2. A(-3, -2), B(14, 4), C(6, 8);

3. A(1, 7), B(-3, -1), C(11, -3);

4. A(1, 0), B(-1, 4), C(9, 5);

5. A(1, -2), B(7, 1), C(6, 1);

6. A(-2, -3), B(1, 6), C(10, 7);

7. A(-4, 2), B(7, 3), C(1, 10);

8. A(4, -3), B(3, 1), C(10, 7);

9. A(4, -4), B(8, 2), C(3, 8);

10. A(-3, -3), B(5, -7), C(7, 7);

11. A(1, -6), B(3, 4), C(-3, 3);

12. A(-4, 2), B(8, -6), C(2, 6);

13. A(-5, 2), B(0, -4), C(5, 7);

14. A(4, -4), B(6, 2), C(-1, 8);

15. A(-3, 8), B(-6, 2), C(0, -5);

16. A(6, -9), B(10, -1), C(-4, 1);

17. A(4, 1), B(-3, -1), C(7, -3);

18. A(-4, 2), B(6, -4), C(4, 10);

19. A(3, -1), B(11, 3), C(-6, 2);

- 20.** A(-7, -2), B(-7, 4), C(5, -5);
21. A(-1, -4), B(9, 6), C(-5, 4);
22. A(10, -2), B(4, -5), C(-3, 1);
23. A(-3, -1), B(-4, -5), C(8, 1);
24. A(-2, -6), B(-3, 5), C(4, 0);
25. A(-7, -2), B(3, -8), C(-4, 6);
26. A(0, 2), B(-7, -4), C(3, 2);
27. A(7, 0), B(1, 4), C(-8, -4);
28. A(-2, 3), B(3, -4), C(10, 2);
29. A(1, -3), B(3, 5), C(6, 2);
30. A(-4, 4), B(2, -1), C(5, 7);
31. A(-6, 1), B(8, 3), C(0, 7);
32. A(1, 4), B(3, -4), C(1, 5);
33. A(-2, -3), B(5, 4), C(1, 7);
34. A(0, 4), B(3, -2), C(5, 6);
35. A(-3, 5), B(4, -1), C(2, 7);
36. A(-4, 3), B(5, 6), C(1, -3);
37. A(2, -4), B(-3, 6), C(0, 5);
38. A(1, -5), B(4, 2), C(6, -1);
39. A(-5, 2), B(6, 4), C(2, -1);
40. A(-4, 5), B(-2, 1), C(5, 3);
41. A(-5, -2), B(3, 2), C(6, 0);
42. A(1, -4), B(8, 1), C(5, 5);
43. A(-6, 3), B(-2, 1), C(2, 6);
44. A(4, 4), B(-3, -1), C(0, -4);
45. A(7, -1), B(3, 1), C(-4, 5);
46. A(6, 1), B(-3, 3), C(2, 5);
47. A(0, 4), B(2, -3), C(-5, 2);
48. A(7, 4), B(-3, 5), C(2, -2);
49. A(7, -1), B(5, 6), C(-4, 3);
50. A(4, -5), B(2, -3), C(-3, 3).

Задача 6. Даны координаты точек

$A_1(x_1, y_1, z_1)$, $A_2(x_2, y_2, z_2)$, $A_3(x_3, y_3, z_3)$, $A_4(x_4, y_4, z_4)$:

Найти:

- 1) уравнение плоскости $(A_1A_2A_3)$;
- 2) уравнение прямой A_1A_2 ;
- 3) уравнение прямой A_4M , перпендикулярной плоскости $(A_1A_2A_3)$;
- 4) уравнение прямой A_3N , параллельная прямой A_1A_2 .

Вычислить:

- 1) объем тетраэдра $A_1A_2A_3A_4$;
- 2) Площадь грани $A_1A_2A_3$ тетраэдра $A_1A_2A_3A_4$;
- 3) Длину высоты опущенной из вершины A_4 на грань $A_1A_2A_3$.

- 1.** $A_1(1, 3, 6), A_2(2, 2, 1), A_3(-1, 0, 1), A_4(-4, 6, -3)$.
- 2.** $A_1(-4, 2, 6), A_2(2, -3, 0), A_3(-1, 5, 8), A_4(-5, 2, -4)$.
- 3.** $A_1(7, 2, 4), A_2(7, -1, -2), A_3(3, 3, 1), A_4(-4, 2, 1)$.
- 4.** $A_1(2, 1, 4), A_2(-1, 5, -2), A_3(-7, -3, 2), A_4(-6, -3, 6)$.
- 5.** $A_1(-1, -5, 2), A_2(-6, 0, -3), A_3(3, 6, -3), A_4(-10, 6, 7)$.
- 6.** $A_1(0, -1, -1), A_2(-2, 3, 5), A_3(1, -5, -9), A_4(-1, -6, 3)$.
- 7.** $A_1(5, 2, 0), A_2(2, 5, 0), A_3(1, 2, 4), A_4(-1, 1, 1)$.
- 8.** $A_1(2, -1, -2), A_2(1, 2, 1), A_3(5, 0, -6), A_4(-10, 9, -7)$.
- 9.** $A_1(-2, 0, -4), A_2(-1, 7, 1), A_3(4, -8, -4), A_4(1, -4, 6)$.
- 10.** $A_1(14, 4, 5), A_2(-5, -3, 2), A_3(-2, -6, -3), A_4(-2, 2, -1)$.
- 11.** $A_1(1, 2, 0), A_2(3, 0, -3), A_3(5, 2, 6), A_4(8, 4, -9)$.
- 12.** $A_1(2, -1, 2), A_2(1, 2, -1), A_3(3, 2, 1), A_4(-4, 2, 5)$.
- 13.** $A_1(1, 1, 2), A_2(-1, 1, 3), A_3(2, -2, 4), A_4(-1, 0, -2)$.
- 14.** $A_1(2, 3, 1), A_2(4, 1, -2), A_3(6, 3, 7), A_4(7, 5, -3)$.

- 15.** $A_1(1, 1, -1), A_2(2, 3, 1), A_3(3, 2, 1), A_4(5, 9, -8)$.
- 16.** $A_1(1, 5, -7), A_2(-3, 6, 3), A_3(-2, 7, 3), A_4(-4, 8, -12)$.
- 17.** $A_1(-3, 4, -7), A_2(1, 5, -4), A_3(-5, -2, 0), A_4(2, 5, 4)$.
- 18.** $A_1(-1, 2, -3), A_2(4, -1, 0), A_3(2, 1, -2), A_4(3, 4, 5)$.
- 19.** $A_1(4, -1, 3), A_2(-2, 1, 0), A_3(0, -5, 1), A_4(3, 2, -6)$.
- 20.** $A_1(1, -1, 1), A_2(-2, 0, 3), A_3(2, 1, -1), A_4(2, -2, -4)$.
- 21.** $A_1(1, 2, 0), A_2(1, -1, 2), A_3(0, 1, -1), A_4(-3, 0, 1)$.
- 22.** $A_1(1, 0, 2), A_2(1, 2, -1), A_3(2, -2, 1), A_4(2, 1, 0)$.
- 23.** $A_1(1, 2, -3), A_2(1, 0, 1), A_3(-2, -1, 6), A_4(0, -5, -4)$.
- 24.** $A_1(3, 10, -1), A_2(-2, 3, -5), A_3(-6, 0, -3), A_4(1, -1, 2)$.
- 25.** $A_1(-1, 2, 4), A_2(-1, -2, -4), A_3(3, 0, -1), A_4(7, -3, 1)$.
- 26.** $A_1(0, -3, 1), A_2(-4, 1, 2), A_3(2, -1, 5), A_4(3, 1, -4)$.
- 27.** $A_1(1, 3, 0), A_2(4, -1, 2), A_3(3, 0, 1), A_4(-4, 3, 5)$.
- 28.** $A_1(-2, -1, -1), A_2(0, 3, 2), A_3(3, 1, -4), A_4(-4, 7, 3)$.
- 29.** $A_1(-3, -5, 6), A_2(2, 1, -4), A_3(0, -3, -1), A_4(-5, 2, -8)$.
- 30.** $A_1(2, 4, -3), A_2(5, -6, 0), A_3(-1, 3, 3), A_4(-10, -8, 7)$.
- 31.** $A_1(0, -1, 5), A_2(-1, 3, 5), A_3(-1, -5, -9), A_4(-1, 7, 3)$.
- 32.** $A_1(6, 6, 5), A_2(4, 9, 5), A_3(4, 6, 11), A_4(6, 9, 3)$.
- 33.** $A_1(7, 2, 2), A_2(-5, 7, -7), A_3(5, -3, 1), A_4(2, 3, 7)$.
- 34.** $A_1(8, -6, 4), A_2(10, 5, -5), A_3(5, 6, -8), A_4(8, 10, 7)$.
- 35.** $A_1(1, -1, 3), A_2(6, 5, 8), A_3(3, 5, 8), A_4(8, 4, 1)$.
- 36.** $A_1(1, -2, 7), A_2(4, 2, 10), A_3(2, 3, 5), A_4(5, 7, 3)$.
- 37.** $A_1(0, 4, 5), A_2(3, -2, 1), A_3(4, 5, 6), A_4(3, 3, 2)$.
- 38.** $A_1(2, -1, 7), A_2(6, 3, 1), A_3(3, 2, 8), A_4(2, -3, 7)$.
- 39.** $A_1(2, 1, 7), A_2(3, 3, 6), A_3(2, -3, 9), A_4(1, 2, 5)$.

- 40.** $A_1(2, 1, 6), A_2(1, 4, 9), A_3(2, -5, 8), A_4(5, 4, 2)$.
- 41.** $A_1(3, 2, 5), A_2(4, 0, 6), A_3(2, 6, 5), A_4(6, 4, -1)$.
- 42.** $A_1(4, 3, 5), A_2(1, 9, 7), A_3(0, 2, 0), A_4(5, 3, 10)$.
- 43.** $A_1(5, 3, 7), A_2(-2, 3, 5), A_3(4, 2, 10), A_4(1, 2, 7)$.
- 44.** $A_1(2, 3, 5), A_2(5, 3, -7), A_3(1, -2, -9), A_4(4, 2, 0)$.
- 45.** $A_1(4, 2, 10), A_2(1, 2, 0), A_3(3, 5, 7), A_4(2, -3, 5)$.
- 46.** $A_1(-3, 2, 0), A_2(4, -2, 7), A_3(2, -6, 5), A_4(3, -4, -1)$.
- 47.** $A_1(2, 3, -5), A_2(-1, 9, -6), A_3(0, 2, 5), A_4(5, 3, -4)$.
- 48.** $A_1(-5, -3, 1), A_2(-2, 3, 6), A_3(4, 2, -4), A_4(1, -2, 6)$.
- 49.** $A_1(1, 6, -5), A_2(2, 3, -1), A_3(1, -2, 7), A_4(3, 2, 5)$.
- 50.** $A_1(1, -2, 9), A_2(4, 3, 0), A_3(-2, 5, 6), A_4(2, -3, 4)$.

Задача 7. Найти:

- 1) угол между плоскостями π_1 и π_2 ;
- 2) уравнение плоскости, проходящей через начало координат, перпендикулярно заданным плоскостям;
- 3) проекцию начала координат на плоскость π_1 .

1. $\pi_1: x - 3y + 5 = 0, \pi_2: 2x - y + 5z - 16 = 0$.
2. $\pi_1: x - 3y + z - 1 = 0, \pi_2: x + z - 1 = 0$.
3. $\pi_1: 4x - 5y + 3z - 1 = 0, \pi_2: x - 4y - z + 9 = 0$.
4. $\pi_1: 3x - y + 2z + 15 = 0, \pi_2: 5x + 9y - 3z - 1 = 0$.
5. $\pi_1: 6x + 2y - 4z + 17 = 0, \pi_2: 9x + 3y - 6z - 4 = 0$.
6. $\pi_1: x - y\sqrt{2} + z - 1 = 0, \pi_2: x + y\sqrt{2} - z + 3 = 0$.
7. $\pi_1: 3y - z = 0 = 0, \pi_2: 2x + z = 0$.
8. $\pi_1: 6x + 3y - 2z = 0, \pi_2: x + 2y + 6z - 12 = 0$.
9. $\pi_1: x + 2y + 2z - 3 = 0, \pi_2: 16x + 12y - 15z - 1 = 0$.

- 10.** $\pi_1 : 2x - y + 5z + 16 = 0$, $\pi_2 : x + 2y + 3z + 8 = 0$.
- 11.** $\pi_1 : x + z - 1 = 0$, $\pi_2 : 2x + 2y + z - 1 = 0$.
- 12.** $\pi_1 : 3x + y + z - 4 = 0$, $\pi_2 : y + z + 5 = 0$.
- 13.** $\pi_1 : 3x - 2y - 2z - 16 = 0$, $\pi_2 : x + y - 3z - 7 = 0$.
- 14.** $\pi_1 : 2x + 2y + z + 9 = 0$, $\pi_2 : x - y + 3z - 1 = 0$.
- 15.** $\pi_1 : x + 2y + 2z - 3 = 0$, $\pi_2 : 2x - y + 2z + 5 = 0$.
- 16.** $\pi_1 : x + y + z - 7 = 0$, $\pi_2 : 3x + 2y - 3z - 1 = 0$.
- 17.** $\pi_1 : x - 3y - 2z - 8 = 0$, $\pi_2 : x + y - z + 3 = 0$.
- 18.** $\pi_1 : 3x - 2y + 3z + 23 = 0$, $\pi_2 : y + z + 5 = 0$.
- 19.** $\pi_1 : x + y + 3z - 7 = 0$, $\pi_2 : y + z - 1 = 0$.
- 20.** $\pi_1 : x - 2y + 2z + 17 = 0$, $\pi_2 : x - 2y - 1 = 0$.
- 21.** $\pi_1 : x + 2y - 1 = 0$, $\pi_2 : x + z + 6 = 0$.
- 22.** $\pi_1 : 2x - z + 5 = 0$, $\pi_2 : 2x + 3y - 7 = 0$.
- 23.** $\pi_1 : 5x + 3y + z - 18 = 0$, $\pi_2 : 2y + z - 9 = 0$.
- 24.** $\pi_1 : x + 2y + 2z + 5 = 0$, $\pi_2 : 4x + 3z - 2 = 0$.
- 25.** $\pi_1 : x + 4y - z + 1 = 0$, $\pi_2 : 2x + y + 4z - 3 = 0$.
- 26.** $\pi_1 : 2y + z - 9 = 0$, $\pi_2 : x - y + 2z - 1 = 0$.
- 27.** $\pi_1 : 2x - 6y + 14z - 1 = 0$, $\pi_2 : 5x - 15y + 35z - 3 = 0$.
- 28.** $\pi_1 : x - y + 7z - 1 = 0$, $\pi_2 : 2x - 2y - 5 = 0$.
- 29.** $\pi_1 : 2x + y - 3 = 0$, $\pi_2 : 3x - y - 5 = 0$.
- 30.** $\pi_1 : x + y + z\sqrt{2} - 3 = 0$, $\pi_2 : x - y + z\sqrt{2} - 1 = 0$.
- 31.** $\pi_1 : x - 3y + 2z + 2 = 0$, $\pi_2 : x + 3y + z + 14 = 0$.
- 32.** $\pi_1 : x + y + z - 2 = 0$, $\pi_2 : x - y - 2z + 2 = 0$.
- 33.** $\pi_1 : 5x + y - 3z + 4 = 0$, $\pi_2 : x - y + 2z + 2 = 0$.
- 34.** $\pi_1 : x + 5y + 2z + 11 = 0$, $\pi_2 : x - y - z - 1 = 0$.
- 35.** $\pi_1 : 4x + y - 3z + 2 = 0$, $\pi_2 : 2x - y + z - 8 = 0$.

- 36.** $\pi_1 : 3x + 3y - 2z - 1 = 0$, $\pi_2 : 2x - 3y + z + 6 = 0$.
- 37.** $\pi_1 : 8x - y - 3z - 1 = 0$, $\pi_2 : x + y + z + 10 = 0$.
- 38.** $\pi_1 : 4x + y + z + 2 = 0$, $\pi_2 : 2x - y - 3z - 8 = 0$.
- 39.** $\pi_1 : x + 5y - z + 11 = 0$, $\pi_2 : x - y + 2z - 1 = 0$.
- 40.** $\pi_1 : 6x - 7y - z - 2 = 0$, $\pi_2 : x + 7y - 4z - 5 = 0$.
- 41.** $\pi_1 : 2x + 3y - 2z + 6 = 0$, $\pi_2 : x - 3y + z + 3 = 0$.
- 42.** $\pi_1 : 2x + 3y - 2z + 6 = 0$, $\pi_2 : x - 3y + z + 3 = 0$.
- 43.** $\pi_1 : 6x - 5y + 3z + 8 = 0$, $\pi_2 : 6x + 5y - 4z + 4 = 0$.
- 44.** $\pi_1 : x + 5y - z - 5 = 0$, $\pi_2 : 2x - 5y + 2z + 5 = 0$.
- 45.** $\pi_1 : x + y - 2z - 2 = 0$, $\pi_2 : x - y + z + 2 = 0$.
- 46.** $\pi_1 : x - 3y - 3z + 5 = 0$, $\pi_2 : x - 2y + 5z + 13 = 0$.
- 47.** $\pi_1 : -2x + 6y - 5z + 16 = 0$, $\pi_2 : 3x - 7y + 9z - 23 = 0$.
- 48.** $\pi_1 : 3x - 8y + 4z - 8 = 0$, $\pi_2 : 4x + y - 6z + 14 = 0$.
- 49.** $\pi_1 : x + 3y - 7z - 15 = 0$, $\pi_2 : -2x + 3y + 4z + 12 = 0$.
- 50.** $\pi_1 : x + 5y - 4z - 21 = 0$, $\pi_2 : 5x - 2y + 6z + 12 = 0$.

Задача 8. Найти:

- 1) канонические уравнения прямой ℓ ;
- 2) уравнение прямой, проходящей через начало координат, перпендикулярно данной прямой и пересекающая её.

$$1. \ell : \begin{cases} 2x + y + z - 2 = 0 \\ 2x - y - 3z + 6 = 0 \end{cases}$$

$$3. \ell : \begin{cases} x + 3y + z + 14 = 0 \\ x - 2y + z - 4 = 0 \end{cases}$$

$$5. \ell : \begin{cases} 2x + 3y + z + 6 = 0 \\ x - 3y - 2z + 3 = 0 \end{cases}$$

$$2. \ell : \begin{cases} x - 3y + 2z + 2 = 0 \\ x + 3y + z + 14 = 0 \end{cases}$$

$$4. \ell : \begin{cases} x + y + z - 2 = 0 \\ x - y - 2z + 2 = 0 \end{cases}$$

$$6. \ell : \begin{cases} 3x + y - z - 6 = 0 \\ 3x - y + 2z = 0 \end{cases}$$

$$7. \ell : \begin{cases} x + 5y + 2z + 11 = 0 \\ x - y - z - 1 = 0 \end{cases}$$

$$9. \ell : \begin{cases} 5x + y - 3z + 4 = 0 \\ x - y + 2z + 2 = 0 \end{cases}$$

$$11. \ell : \begin{cases} 4x + y - 3z + 2 = 0 \\ 2x - y + z - 8 = 0 \end{cases}$$

$$13. \ell : \begin{cases} x + 7y - z - 5 = 0 \\ 6x - 7y - 4z - 2 = 0 \end{cases}$$

$$15. \ell : \begin{cases} 6x - 5y - 4z + 8 = 0 \\ 6x + 5y + 3z + 4 = 0 \end{cases}$$

$$17. \ell : \begin{cases} x + 5y - z - 5 = 0 \\ 2x - 5y + 2z + 5 = 0 \end{cases}$$

$$19. \ell : \begin{cases} 4x + y + z + 2 = 0 \\ 2x - y - 3z - 8 = 0 \end{cases}$$

$$21. \ell : \begin{cases} x + y - 2z - 2 = 0 \\ x - y + z + 2 = 0 \end{cases}$$

$$23. \ell : \begin{cases} x - y + z - 2 = 0 \\ x - 2y - z + 4 = 0 \end{cases}$$

$$25. \ell : \begin{cases} x + 5y + 2z - 5 = 0 \\ 2x - 5y - z + 5 = 0 \end{cases}$$

$$27. \ell : \begin{cases} 2x + 3y - 2z + 6 = 0 \\ x - 3y + z + 3 = 0 \end{cases}$$

$$29. \ell : \begin{cases} 3x + 3y + z - 1 = 0 \\ 2x - 3y - 2z + 6 = 0 \end{cases}$$

$$31. \ell : \begin{cases} 6x + 2y - 4z + 17 = 0 \\ 9x + 3y - 6z - 4 = 0 \end{cases}$$

$$33. \ell : \begin{cases} 6x + 3y - 2z = 0 \\ x + 2y + 6z - 12 = 0 \end{cases}$$

$$8. \ell : \begin{cases} 3x + 4y - 2z + 1 = 0 \\ 2x - 4y + 3z + 4 = 0 \end{cases}$$

$$10. \ell : \begin{cases} x - y - z - 2 = 0 \\ x - 2y + z + 4 = 0 \end{cases}$$

$$12. \ell : \begin{cases} 3x + 3y - 2z - 1 = 0 \\ 2x - 3y + z + 6 = 0 \end{cases}$$

$$14. \ell : \begin{cases} 8x - y - 3z - 1 = 0 \\ x + y + z + 10 = 0 \end{cases}$$

$$16. \ell : \begin{cases} x + 5y - z - 5 = 0 \\ 2x - 5y + 2z + 5 = 0 \end{cases}$$

$$18. \ell : \begin{cases} x - y - 3z + 2 = 0 \\ 5x + y + 2z + 4 = 0 \end{cases}$$

$$20. \ell : \begin{cases} 2x + y - 3z - 2 = 0 \\ 2x - y + z + 6 = 0 \end{cases}$$

$$22. \ell : \begin{cases} x + 5y - z + 11 = 0 \\ x - y + 2z - 1 = 0 \end{cases}$$

$$24. \ell : \begin{cases} 6x - 7y - z - 2 = 0 \\ x + 7y - 4z - 5 = 0 \end{cases}$$

$$26. \ell : \begin{cases} x - 3y + z + 2 = 0 \\ x + 3y + 2z + 14 = 0 \end{cases}$$

$$28. \ell : \begin{cases} 3x + 4y + 3z + 1 = 0 \\ 2x - 4y - 2z + 4 = 0 \end{cases}$$

$$30. \ell : \begin{cases} 6x - 5y + 3z + 8 = 0 \\ 6x + 5y - 4z + 4 = 0 \end{cases}$$

$$32. \ell : \begin{cases} 3x - y + 2z + 15 = 0 \\ 5x + 9y - 3z - 1 = 0 \end{cases}$$

$$34. \ell : \begin{cases} 2x - y + 5z + 16 = 0 \\ x + 2y + 3z + 8 = 0 \end{cases}$$

$$35. \ell : \begin{cases} 3x + y + z - 4 = 0 \\ y + z + 5 = 0 \end{cases}.$$

$$37. \ell : \begin{cases} x + 2y + 2z - 3 = 0 \\ 2x - y + 2z + 5 = 0 \end{cases}.$$

$$39. \ell : \begin{cases} x + 2y - 1 = 0 \\ x + y + 6 = 0 \end{cases}.$$

$$41. \ell : \begin{cases} x + 4y - z + 1 = 0 \\ 2x + y + 4z - 3 = 0 \end{cases}.$$

$$43. \ell : \begin{cases} 6x + 2y - 4z + 17 = 0 \\ 9x + 3y - 6z - 4 = 0 \end{cases}.$$

$$45. \ell : \begin{cases} 3x - 2y - 2z - 16 = 0 \\ x + y - 3z - 7 = 0 \end{cases}.$$

$$47. \ell : \begin{cases} 3x - 2y + 6z - 10 = 0 \\ 3x + y + 2z - 15 = 0 \end{cases}.$$

$$49. \ell : \begin{cases} -x + 3y - 6z + 19 = 0 \\ 2x - 3y + 5z - 14 = 0 \end{cases}.$$

$$36. \ell : \begin{cases} 2x + 2y + z + 9 = 0 \\ x - y + 3z - 1 = 0 \end{cases}.$$

$$38. \ell : \begin{cases} x + y + 3z - 7 = 0 \\ y + z - 1 = 0 \end{cases}.$$

$$40. \ell : \begin{cases} 5x + 3y + z - 18 = 0 \\ 2y + z - 9 = 0 \end{cases}.$$

$$42. \ell : \begin{cases} x - y + 7z - 1 = 0 \\ 2x - 2y - 5 = 0 \end{cases}.$$

$$44. \ell : \begin{cases} x - y + 7z - 1 = 0 \\ 2x - y - 5 = 0 \end{cases}.$$

$$46. \ell : \begin{cases} 2x - 3y + z + 11 = 0 \\ x + y - 5z - 8 = 0 \end{cases}.$$

$$48. \ell : \begin{cases} 4x - 2y + z - 21 = 0 \\ 3x + 7y - 5z + 3 = 0 \end{cases}.$$

$$50. \ell : \begin{cases} 4x - 5y + 2z - 6 = 0 \\ 3x + y - 5z - 10 = 0 \end{cases}.$$

Задача 9. Найти:

- 1) координаты точки пересечения прямой ℓ с плоскостью π ;
- 2) канонические уравнения проекции прямой ℓ на плоскости π .

$$1. \ell : \frac{x-2}{-1} = \frac{y-3}{-1} = \frac{z+1}{4}, \pi: x + 2y + 3z - 14 = 0.$$

$$2. \ell : \frac{x+1}{3} = \frac{y-3}{-4} = \frac{z+1}{5}, \pi: x + 2y - 5z + 20 = 0.$$

$$3. \ell: \frac{x-1}{-1} = \frac{y+5}{4} = \frac{z-1}{2}, \pi: x - 3y + 7z - 24 = 0.$$

$$4. \ell: \frac{x-1}{1} = \frac{y}{0} = \frac{z+3}{2}, \pi: 2x - y + 4z = 0.$$

$$5. \ell: \frac{x-5}{1} = \frac{y-3}{-1} = \frac{z-2}{0}, \pi: 3x + y - 5z - 12 = 0.$$

$$6. \ell: \frac{x+1}{-3} = \frac{y+2}{2} = \frac{z-3}{-2}, \pi: x + 3y - 5z + 9 = 0.$$

$$7. \ell: \frac{x-1}{-2} = \frac{y-2}{1} = \frac{z+1}{-1}, \pi: x - 2y + 5z + 17 = 0.$$

$$8. \ell: \frac{x-1}{2} = \frac{y-2}{0} = \frac{z-4}{1}, \pi: x - 2y + 4z - 19 = 0.$$

$$9. \ell: \frac{x+2}{-1} = \frac{y-1}{1} = \frac{z+4}{-1}, \pi: 2x - y + 3z + 23 = 0.$$

$$10. \ell: \frac{x+2}{1} = \frac{y-2}{0} = \frac{z+3}{0}, \pi: 2x - 3y - 5z - 7 = 0.$$

$$11. \ell: \frac{x-1}{2} = \frac{y-1}{-1} = \frac{z+2}{3}, \pi: 4x + 2y - z - 11 = 0.$$

$$12. \ell: \frac{x-1}{1} = \frac{y+1}{0} = \frac{z-1}{-1}, \pi: 3x - 2y - 4z - 8 = 0.$$

$$13. \ell: \frac{x+2}{-1} = \frac{y-1}{1} = \frac{z+3}{2}, \pi: x + 2y - z - 2 = 0.$$

$$14. \ell: \frac{x+3}{1} = \frac{y-2}{-5} = \frac{z+2}{3}, \pi: 5x - y + 4z + 3 = 0.$$

$$15. \ell: \frac{x-2}{2} = \frac{y-2}{-1} = \frac{z-4}{3}, \pi: x + 3y + 5z - 42 = 0.$$

$$16. \ell: \frac{x-3}{-1} = \frac{y-4}{5} = \frac{z-4}{2}, \pi: 7x + y + 4z - 47 = 0.$$

$$17. \ell: \frac{x+3}{2} = \frac{y-1}{3} = \frac{z-1}{5}, \pi: 2x + 3y + 7z - 52 = 0.$$

$$18. \ell: \frac{x-3}{2} = \frac{y+1}{3} = \frac{z+3}{2}, \pi: 3x + 4y + 7z - 16 = 0.$$

$$19. \ell: \frac{x-5}{-2} = \frac{y-2}{0} = \frac{z+4}{-1}, \pi: 2x - 5y + 4z + 24 = 0.$$

$$20. \ell: \frac{x-1}{8} = \frac{y-8}{-5} = \frac{z+5}{12}, \pi: x - 2y - 3z + 18 = 0.$$

$$21. \ell: \frac{x-3}{1} = \frac{y-1}{-1} = \frac{z+5}{0}, \pi: x + 7y + 3z + 11 = 0.$$

$$22. \ell: \frac{x-5}{-1} = \frac{y+3}{5} = \frac{z-1}{2}, \pi: 3x + 7y - 5z - 11 = 0.$$

$$23. \ell: \frac{x-1}{7} = \frac{y-2}{1} = \frac{z-6}{-1}, \pi: 4x + y - 6z - 5 = 0.$$

$$24. \ell: \frac{x-3}{1} = \frac{y+2}{-1} = \frac{z-8}{0}, \pi: 5x + 9y + 4z - 25 = 0.$$

$$25. \ell: \frac{x+1}{-2} = \frac{y}{0} = \frac{z+1}{3}, \pi: x + 4y + 13z - 23 = 0.$$

$$26. \ell: \frac{x-1}{6} = \frac{y-3}{1} = \frac{z+5}{3}, \pi: 3x - 2y + 5z - 3 = 0.$$

$$27. \ell: \frac{x-2}{4} = \frac{y-1}{-3} = \frac{z+3}{-2}, \pi: 3x - y + 4z = 0.$$

$$28. \ell: \frac{x-1}{2} = \frac{y+2}{-5} = \frac{z-3}{-2}, \pi: x + 2y - 5z + 16 = 0.$$

$$29. \ell: \frac{x-1}{1} = \frac{y-3}{0} = \frac{z+2}{-2}, \pi: 3x - 7y - 2z + 7 = 0.$$

$$30. \ell: \frac{x+3}{0} = \frac{y-2}{-3} = \frac{z+5}{11}, \pi: 5x + 7y + 9z - 32 = 0.$$

$$31. \ell: \frac{x+2}{-1} = \frac{y-1}{1} = \frac{z+3}{2}, \pi: x + 2y - z - 2 = 0.$$

$$32. \ell: \frac{x-7}{3} = \frac{y-3}{1} = \frac{z+1}{-2}, \pi: 2x + y + 7z - 3 = 0.$$

$$33. \ell: \frac{x-2}{5} = \frac{y-3}{1} = \frac{z+1}{2}, \pi: 11x - 17y - 19z + 10 = 0.$$

$$34. \ell: \frac{x+2}{2} = \frac{y-3}{4} = \frac{z}{3}, \pi: 3x - y + 2z - 4 = 0.$$

$$35. \ell: \frac{x-10}{8} = \frac{y-3}{2} = \frac{z-4}{3}, \pi: x+2y-4z+1=0.$$

$$36. \ell: \frac{x-7}{5} = \frac{y-4}{1} = \frac{z-5}{4}, \pi: 3x-y+2z-5=0.$$

$$37. \ell: \frac{x}{1} = \frac{y-1}{-2} = \frac{z}{3}, \pi: 3x+y-5z+1=0.$$

$$38. \ell: \frac{x-7}{5} = \frac{y-1}{1} = \frac{z-5}{4}, \pi: 3x-y+2z-8=0.$$

$$39. \ell: \frac{x+3}{-1} = \frac{y-2}{0} = \frac{z+1}{2}, \pi: 2x+2y-3z+4=0.$$

$$40. \ell: \frac{x+4}{-2} = \frac{y-1}{1} = \frac{z-2}{3}, \pi: x+2y-z+3=0.$$

$$41. \ell: \frac{x+1}{-3} = \frac{y+3}{-2} = \frac{z+1}{1}, \pi: x+2y-z+5=0.$$

$$42. \ell: \frac{x+1}{-3} = \frac{y-3}{1} = \frac{z-1}{-2}, \pi: -x+2y-z-3=0.$$

$$43. \ell: \frac{x-2}{3} = \frac{y+1}{-2} = \frac{z}{2}, \pi: 4x+y-3z+2=0.$$

$$44. \ell: \frac{x+5}{-3} = \frac{y-3}{1} = \frac{z+3}{7}, \pi: x+2y-5z+3=0.$$

$$45. \ell: \frac{x+1}{-3} = \frac{y+1}{5} = \frac{z-2}{2}, \pi: 3x+y-3z+4=0.$$

$$46. \ell: \frac{x-1}{3} = \frac{y+2}{2} = \frac{z+1}{-1}, \pi: 2x-y-3z+15=0.$$

$$47. \ell: \frac{x-1}{3} = \frac{y+3}{-1} = \frac{z-2}{2}, \pi: -2x+y-3z-13=0.$$

$$48. \ell: \frac{x+2}{-3} = \frac{y-3}{2} = \frac{z-4}{-2}, \pi: x-y+2z-7=0.$$

$$49. \ell: \frac{x-5}{3} = \frac{y+3}{-1} = \frac{z-2}{5}, \pi: 2x-3y+5z+12=0.$$

$$50. \ell: \frac{x-2}{3} = \frac{y+1}{4} = \frac{z-3}{-2}, \pi: 2x-4y-z+4=0.$$

Задача 10. Исследовать на линейную зависимость систему векторов.

1. $\vec{a} = \{1, -4, 6\}$, $\vec{b} = \{1, -1, 1\}$, $\vec{c} = \{1, 1, 3\}$.

2. $\sin x$, $\cos x$, $\operatorname{tg} x$ на $(-\pi/2, \pi/2)$.

3. $\vec{a} = \{2, -3, 1\}$, $\vec{b} = \{3, -1, 5\}$, $\vec{c} = \{1, -4, 3\}$.

4. 2 , $\sin x$, $\sin^2 x$, $\cos^2 x$ на $(-\infty, +\infty)$.

5. $\vec{a} = \{5, 4, 3\}$, $\vec{b} = \{3, 3, 2\}$, $\vec{c} = \{8, 1, 3\}$.

6. 1 , x , $\sin x$ на $(-\infty, +\infty)$.

7. $\vec{a} = \{1, 1, 1\}$, $\vec{b} = \{0, 1, 1\}$, $\vec{c} = \{0, 0, 1\}$.

8. e^x , e^{2x} , e^{3x} на $(-\infty, +\infty)$.

9. $\vec{a} = \{1, -1, 2\}$, $\vec{b} = \{-1, 1, -1\}$, $\vec{c} = \{2, -1, 1\}$.

10. x , x^2 , $(1+x)^2$ на $(-\infty, +\infty)$.

11. $\vec{a} = \{1, 2, 3\}$, $\vec{b} = \{4, 5, 6\}$, $\vec{c} = \{7, 8, 9\}$.

12. 1 , x , x^2 , $(1+x)^2$ на $(-\infty, +\infty)$.

13. $\vec{a} = \{1, 1, 1\}$, $\vec{b} = \{1, 2, 3\}$, $\vec{c} = \{1, 3, 6\}$.

14. $\cos x$, $\sin x$, $\sin 2x$ на $(-\pi/2, \pi/2)$.

15. $\vec{a} = \{3, 4, -5\}$, $\vec{b} = \{8, 7, -2\}$, $\vec{c} = \{2, -1, -8\}$.

$$16. e^x, e^{-x}, e^{2x} \text{ ha } (-\infty, +\infty).$$

$$17. \vec{a} = \{3, 2, -4\}, \vec{b} = \{4, 1, -2\}, \vec{c} = \{5, 2, -3\}.$$

$$18. 1+x+x^2, 1+2x+x^2, 1+3x+x^2 \text{ ha } (-\infty, +\infty).$$

$$19. \vec{a} = \{0, 1, 1\}, \vec{b} = \{1, 0, 1\}, \vec{c} = \{1, 1, 0\}.$$

$$20. 1, e^x, \operatorname{sh} x \text{ ha } (-\infty, +\infty).$$

$$21. \vec{a} = \{5, -6, 1\}, \vec{b} = \{3, -5, -2\}, \vec{c} = \{2, -1, 3\}.$$

$$22. \frac{1}{x}, x, 1 \text{ ha } (0, 1).$$

$$23. \vec{a} = \{7, 1, -3\}, \vec{b} = \{2, 2, -4\}, \vec{c} = \{3, -3, 5\}.$$

$$24. 1, \operatorname{tg} x, \operatorname{ctg} x \text{ ha } (0, \pi/2).$$

$$25. \vec{a} = \{1, 2, 3\}, \vec{b} = \{6, 5, 9\}, \vec{c} = \{7, 8, 9\}.$$

$$26. x, 1+x, (1+x)^2 \text{ ha } (-\infty, +\infty).$$

$$27. \vec{a} = \{2, 1, 0\}, \vec{b} = \{-5, 0, 3\}, \vec{c} = \{3, 4, 3\}.$$

$$28. e^x, xe^x, x^2e^x \text{ ha } (-\infty, +\infty).$$

$$29. \vec{a} = \{2, 0, 2\}, \vec{b} = \{1, -1, 0\}, \vec{c} = \{0, -1, -2\}.$$

$$30. e^x, \operatorname{sh} x, \operatorname{ch} x \text{ ha } (-\infty, +\infty).$$

$$31. \vec{a} = \{-2, 1, 5\}, \vec{b} = \{4, -3, 0\}, \vec{c} = \{0, -1, 10\}.$$

$$32. 1, \sin^2 x, \cos^2 x \text{ на } (-\infty, +\infty).$$

$$33. \vec{a} = \{3, -2, 1\}, \vec{b} = \{4, 5, -2\}, \vec{c} = \{-1, -30, 13\}.$$

$$34. 1, \sin x, \cos x, \sin 2x, \cos 2x \text{ на } (-\infty, +\infty).$$

$$35. \vec{a} = \{1, 0, -3\}, \vec{b} = \{0, 4, -5\}, \vec{c} = \{7, -1, 0\}.$$

$$36. 1, \cos x, \sin x, \cos^2 x, \sin^2 x, \dots, \cos^n x, \sin^n x \text{ на } (-\infty, +\infty).$$

$$37. \vec{a} = \{1, 2, 3\}, \vec{b} = \{2, 3, 1\}, \vec{c} = \{3, 1, 2\}.$$

$$38. 1, \sin^2 x, \cos 2x \text{ на } (-\infty, +\infty).$$

$$39. \vec{a} = \{1, 3, -1\}, \vec{b} = \{0, -2, 1\}, \vec{c} = \{-3, -1, -1\}.$$

$$40. 1, x, x^2, (1-x)^2 \text{ на } (-\infty, +\infty).$$

$$41. \vec{a} = \{1, 1, 1\}, \vec{b} = \{0, 1, 0\}, \vec{c} = \{0, 0, 1\}.$$

$$42. 1, x, x^2, x^3, (1+x)^3 \text{ на } (-\infty, +\infty).$$

$$43. \vec{a} = \{1, 1, 1\}, \vec{b} = \{1, 1, 2\}, \vec{c} = \{0, 0, 1\}.$$

$$44. e^{\alpha x} \cos \beta x, e^{\alpha x} \sin \beta x \text{ на } (-\infty, +\infty).$$

$$45. \vec{a} = \{3, 4, -5\}, \vec{b} = \{8, 7, -2\}, \vec{c} = \{-5, -3, -3\}.$$

$$46. e^x, e^{-x}, e^{-2x} \text{ на } (-\infty, +\infty).$$

$$47. \vec{a} = \{3, 2, -4\}, \vec{b} = \{4, 1, -2\}, \vec{c} = \{10, 5, -10\}.$$

48. $1, x, 3x^2, x^3, (1+x)^3$ на $(-\infty, +\infty)$.

49. $\vec{a} = \{0, 1, 1\}, \vec{b} = \{1, 0, 1\}, \vec{c} = \{-2, 1, -1\}$.

50. $1, \cos 2x, \cos^2 x$ на $(-\infty, +\infty)$.

Задача 11. Найти собственные значения и собственные векторы матрицы.

1. $\begin{pmatrix} 4 & -2 & -1 \\ -1 & 3 & -1 \\ 1 & -2 & 2 \end{pmatrix};$ 2. $\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 1 & -1 & 1 \end{pmatrix};$ 3. $\begin{pmatrix} 3 & -1 & 1 \\ 0 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix};$

4. $\begin{pmatrix} 5 & -1 & -1 \\ 0 & 4 & -1 \\ 0 & -1 & 4 \end{pmatrix};$ 5. $\begin{pmatrix} 6 & -2 & -1 \\ -1 & 5 & -1 \\ 1 & -2 & 4 \end{pmatrix};$ 6. $\begin{pmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ -2 & 1 & 4 \end{pmatrix};$

7. $\begin{pmatrix} 2 & 0 & -1 \\ 1 & 1 & -1 \\ -1 & 0 & 2 \end{pmatrix};$ 8. $\begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ -1 & 1 & 3 \end{pmatrix};$ 9. $\begin{pmatrix} 4 & 1 & 0 \\ 1 & 4 & 0 \\ -1 & 1 & 5 \end{pmatrix};$

10. $\begin{pmatrix} 5 & 1 & -1 \\ 2 & 4 & -1 \\ -2 & 1 & 6 \end{pmatrix};$ 11. $\begin{pmatrix} 5 & -4 & 4 \\ 2 & 1 & 2 \\ 2 & 0 & 3 \end{pmatrix};$ 12. $\begin{pmatrix} 3 & -2 & 2 \\ 2 & -1 & 2 \\ 2 & -2 & 3 \end{pmatrix};$

13. $\begin{pmatrix} 3 & -2 & 2 \\ 0 & 3 & 0 \\ 0 & 2 & 1 \end{pmatrix};$ 14. $\begin{pmatrix} 5 & -2 & 2 \\ 0 & 5 & 0 \\ 0 & 2 & 3 \end{pmatrix};$ 15. $\begin{pmatrix} 7 & -4 & 4 \\ 2 & 3 & 2 \\ 2 & 0 & 5 \end{pmatrix};$

$$16. \begin{pmatrix} 7 & -6 & 6 \\ 4 & -1 & 4 \\ 4 & -2 & 5 \end{pmatrix}; 17. \begin{pmatrix} 7 & -6 & 6 \\ 2 & 3 & 2 \\ 2 & 2 & 3 \end{pmatrix}; 18. \begin{pmatrix} 13 & 2 & -2 \\ 6 & 9 & -6 \\ 2 & -2 & 5 \end{pmatrix};$$

$$19. \begin{pmatrix} \frac{7}{3} & \frac{2}{3} & -\frac{2}{3} \\ \frac{4}{3} & \frac{5}{3} & -\frac{2}{3} \\ 0 & 0 & 1 \end{pmatrix}; 20. \begin{pmatrix} 3 & 0 & 0 \\ \frac{2}{3} & \frac{7}{3} & -\frac{4}{3} \\ \frac{2}{3} & -\frac{2}{3} & \frac{5}{3} \end{pmatrix}; 21. \begin{pmatrix} 5 & 0 & 0 \\ \frac{1}{3} & \frac{13}{3} & -\frac{4}{3} \\ \frac{2}{3} & -\frac{2}{3} & \frac{11}{3} \end{pmatrix};$$

$$22. \begin{pmatrix} 19/3 & 2/3 & -2/3 \\ 2 & 5 & -2 \\ 2/3 & -2/3 & 11/3 \end{pmatrix}; 23. \begin{pmatrix} 4 & 1 & -1 \\ 2 & 3 & -2 \\ 1 & -1 & 2 \end{pmatrix}; 24. \begin{pmatrix} 2 & 1 & -1 \\ 1 & 2 & -1 \\ 0 & 0 & 1 \end{pmatrix};$$

$$25. \begin{pmatrix} 3 & 0 & 0 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}; 26. \begin{pmatrix} 5 & 0 & 0 \\ 1 & 4 & -1 \\ 1 & -1 & 4 \end{pmatrix}; 27. \begin{pmatrix} 6 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & -1 & 4 \end{pmatrix};$$

$$28. \begin{pmatrix} 3 & -2 & -2 \\ -2/3 & 5/3 & -2/3 \\ -2/3 & 2/3 & -13/3 \end{pmatrix}; 29. \begin{pmatrix} 5/3 & -2/3 & -4/3 \\ 0 & 1 & 0 \\ -2/3 & 2/3 & 7/3 \end{pmatrix}; 30. \begin{pmatrix} 7 & -4 & -2 \\ -2 & 5 & -2 \\ 0 & 0 & 9 \end{pmatrix};$$

$$31. \begin{pmatrix} 4 & -3 & -3 \\ 1 & 2 & 1 \\ -1 & 1 & 2 \end{pmatrix}; 32. \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}; 33. \begin{pmatrix} 1 & -2 & -2 \\ 1 & 4 & 2 \\ 0 & -6 & -4 \end{pmatrix};$$

$$34. \begin{pmatrix} 1 & 0 & 1 \\ 1 & 2 & 0 \\ 8 & 0 & -1 \end{pmatrix}; \quad 35. \begin{pmatrix} 1 & 0 & 3 \\ -2 & 3 & -2 \\ 0 & 0 & -2 \end{pmatrix}; \quad 36. \begin{pmatrix} 7 & 2 & -2 \\ 4 & 5 & -2 \\ 0 & 0 & 3 \end{pmatrix};$$

$$37. \begin{pmatrix} 2 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 2 \end{pmatrix}; \quad 38. \begin{pmatrix} 0 & 1 & 2 \\ 4 & 0 & 1 \\ 3 & -1 & 1 \end{pmatrix}; \quad 39. \begin{pmatrix} 5 & 6 & 3 \\ -1 & 0 & 1 \\ 1 & 2 & -1 \end{pmatrix};$$

$$40. \begin{pmatrix} 1 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}; \quad 41. \begin{pmatrix} 5 & -1 & -1 \\ 0 & 4 & -1 \\ 0 & -1 & 4 \end{pmatrix}; \quad 42. \begin{pmatrix} 0 & -1 & 1 \\ -2 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix};$$

$$43. \begin{pmatrix} -2 & 1 & 1 \\ -8 & 4 & 2 \\ -4 & 1 & 3 \end{pmatrix}; \quad 44. \begin{pmatrix} 3 & 1 & 0 \\ -4 & -1 & 0 \\ 4 & -8 & -2 \end{pmatrix}; \quad 45. \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 1 & -1 & 1 \end{pmatrix};$$

$$46. \begin{pmatrix} 1 & 0 & 1 \\ 1 & 2 & 0 \\ 8 & 0 & -1 \end{pmatrix}; \quad 47. \begin{pmatrix} 2 & -1 & -1 \\ -3 & 2 & 0 \\ 4 & 2 & 4 \end{pmatrix}; \quad 48. \begin{pmatrix} 3 & 1 & 0 \\ -4 & -1 & 0 \\ 4 & -8 & -2 \end{pmatrix};$$

$$49. \begin{pmatrix} 7 & 2 & -2 \\ 4 & 5 & -2 \\ 0 & 0 & 3 \end{pmatrix}; \quad 50. \begin{pmatrix} 6 & -5 & -3 \\ 3 & -2 & -2 \\ 2 & -2 & 0 \end{pmatrix}.$$

Задача 12. Исследовать кривую второго порядка и построить ее.

$$1. -x^2 - y^2 + 4xy + 2x - 4y + 1 = 0.$$

$$2. 2x^2 + 2y^2 - 2xy - 2x - 2y + 1 = 0.$$

$$3. 4xy + 4x - 4y = 0.$$

$$4. -2x^2 - 2y^2 + 2xy - 6x + 6y + 3 = 0.$$

$$5. -3x^2 - 3y^2 + 4xy - 6x + 4y + 2 = 0.$$

$$6. -2xy - 2x - 2y + 1 = 0.$$

$$7. -x^2 - y^2 - 4xy - 4x - 2y + 2 = 0.$$

$$8. -4x^2 - 4y^2 + 2xy + 10x - 10y + 1 = 0.$$

$$9. 4xy + 4x - 4y - 2 = 0.$$

$$10. x^2 + y^2 + 2xy - 8x - 8y + 1 = 0.$$

$$11. x^2 + y^2 + 4xy - 8x - 4y + 1 = 0.$$

$$12. x^2 + y^2 - 2xy - 2x + 2y - 7 = 0.$$

$$13. 2xy + 2x + 2y - 3 = 0.$$

$$14. 4x^2 + 4y^2 + 2xy + 12x + 12y + 1 = 0.$$

$$15. 3x^2 + 3y^2 + 4xy + 8x + 12y + 1 = 0.$$

$$16. x^2 + y^2 - 8xy - 20x + 20y + 1 = 0.$$

$$17. 3x^2 + 3y^2 - 2xy - 6x + 2y + 1 = 0.$$

$$18. 4xy + 4x + 4y + 1 = 0.$$

$$19. 3x^2 + 3y^2 - 4xy + 6x - 4y - 7 = 0.$$

$$20. -4xy - 4x + 4y + 6 = 0.$$

$$21. 5x^2 + 5y^2 - 2xy + 10x - 2y + 1 = 0.$$

$$22. 2x^2 + 2y^2 + 4xy + 8x + 8y + 1 = 0.$$

$$23. -x^2 - y^2 + 2xy + 2x - 2y + 1 = 0.$$

$$24. 2x^2 + 2y^2 - 4xy - 8x + 8y + 1 = 0.$$

$$25. 3x^2 + 3y^2 + 2xy - 12x - 4y + 1 = 0.$$

$$26. -4xy + 8x + 8y + 1 = 0.$$

$$27. 2x^2 + 2y^2 - 2xy + 6x - 6y - 6 = 0.$$

$$28. x^2 + y^2 + 4xy + 4x + 2y - 5 = 0.$$

$$29. 4xy + 4x - 4y + 4 = 0.$$

$$30. 3x^2 + 3y^2 - 4xy + 4x + 4y + 1 = 0.$$

$$31. x^2 + y^2 - 4xy + 4x - 2y + 1 = 0.$$

$$32. 3x^2 + 3y^2 + 10xy - 2x - 14y - 13 = 0.$$

$$33. 25x^2 + 25y^2 - 14xy + 64x - 64y - 224 = 0.$$

$$34. 3y^2 + 4xy + 16x + 12y - 36 = 0.$$

$$35. 14x^2 + 21y^2 + 24xy - 4x + 18y - 139 = 0.$$

$$36. 11x^2 - 4y^2 - 20xy - 20x - 8y + 1 = 0.$$

$$37. 29x^2 + 36y^2 - 24xy + 82x - 96y - 91 = 0.$$

$$38. 4x^2 + 11y^2 + 24xy + 64x + 42y + 51 = 0.$$

$$39. 41x^2 + 9y^2 + 24xy + 24x + 18y - 36 = 0.$$

$$40. 9x^2 + 16y^2 - 24xy - 20x + 110y - 50 = 0.$$

$$41. 7x^2 - y^2 + 6xy + 28x + 12y + 28 = 0.$$

$$42. 19x^2 + 11y^2 + 6xy + 38x + 6y + 29 = 0.$$

$$43. 5x^2 + 5y^2 - 2xy - 4x + 20y + 20 = 0.$$

$$44. 7x^2 + 32y^2 + 60xy - 14x - 60y + 7 = 0.$$

$$45. 50x^2 + 35y^2 - 8xy + 100x - 8y + 67 = 0.$$

$$46. 41x^2 + 34y^2 + 24xy + 34x - 112y + 129 = 0.$$

$$47. 9x^2 + 4y^2 + 12xy - 24x - 16y + 3 = 0.$$

$$48. 16x^2 + 9y^2 - 24xy - 160x + 120y + 425 = 0.$$

$$49. \ 3x^2 - 3y^2 + 8xy - 30x + 10y - 25 = 0.$$

$$50. \ 17x^2 + 8y^2 + 12xy - 30x + 60y + 45 = 0.$$