SERII CU TERMENI POZITIVI. CRITERII DE CONVERGENȚĂ

I. Să se cerceteze convergența seriilor $\sum a_n$, folosind criteriul D'Alambert:

1) $a_n = \dfrac{n^4}{(n+1)!}$;

2) $a_n = \dfrac{n^7}{3^n}$;

3) $a_n = \dfrac{2\cdot 5\cdot 8\cdot \ldots \cdot (3n-1)}{1\cdot 6\cdot 11\cdot \ldots \cdot (5n-4)}$;

4) $a_n = \dfrac{1\cdot 3\cdot 5\cdot \ldots \cdot (2n-1)}{3^n \cdot n!}$;

5) $a_n = \dfrac{(2n)!}{(n!)^2}$;

6) $a_n = \dfrac{n^n}{n!\cdot (2,5)^{n+1}}$;

7) $a_n = \dfrac{2\cdot 5\cdot \ldots \cdot (3n+2)}{2^n \cdot (n+1)!}$;

8) $a_n = \dfrac{(2n+1)!!}{3^n \cdot n!}$;

9) $a_n = \arcsin \dfrac{1}{3^n}$;

10) $a_n = \dfrac{3\cdot 6\cdot 9\cdot \ldots \cdot (3n)}{(n+1)!} \arcsin \dfrac{1}{2^n}$;

11) $a_n = \dfrac{(2n)!!}{n!} arctg \dfrac{1}{3^n}$;

12) $a_n = \dfrac{(2n)!}{n!\cdot (n+1)!\cdot 3^{2n}}$;

13) $a_n = \dfrac{7^n}{n\cdot 3^n}$;

14) $a_n = \dfrac{n^n}{n!\cdot 3^n}$;

15) $a_n = \dfrac{3^n}{\sqrt{n^3+1}}$;

16) $a_n = \dfrac{1}{(4n+1)\cdot 5^{2n-1}}$;

17) $a_n = \dfrac{3^{2n}\cdot (n!)^4}{(3n)!\cdot (n+1)!}$.

II. Să se cerceteze convergența seriilor $\sum_{n=0}^{\infty} a_n$, folosind criteriul Cauchy:

1) $a_n = \left(\dfrac{2n+1}{3n-1}\right)^n$;

2) $a_n = \dfrac{1}{(\ln n)^n}$;

3) $a_n = \dfrac{3^n}{n^4}$;

4) $a_n = \dfrac{3^n}{n^n}$;

5) $a_n = 3^n \left(\dfrac{n}{n+1}\right)^{n^2}$;

6) $a_n = \left(\dfrac{4n+1}{2n+3}\right)^{3n}$;

7) $a_n = \left(\dfrac{3n+1}{n+2}\right)^{n^2}$;

8) $a_n = \left(\dfrac{n^2+5}{n^2+6}\right)^{n^3}$;

9) $a_n = \left(\dfrac{n-1}{n+1}\right)^{n^2+4n+5}$;

10) $a_n = \dfrac{1}{3^n}\left(\dfrac{n+2}{n}\right)^{n^2}$;

11) $a_n = \left(n \sin \dfrac{1}{n}\right)^{n^3}$;

12) $a_n = \left(n \arcsin \dfrac{1}{n}\right)^{n^3}$;

13) $a_n = \left(n \arcsin \dfrac{2}{n}\right)^{n^3}$;

14) $a_n = \left(\cos \dfrac{1}{\sqrt{n}}\right)^{n^2}$;

15) $a_n = n^4 \left(\dfrac{3n+2}{4n+5}\right)^n$;

16) $a_n = \left(\dfrac{3n}{n+5}\right)^n$;

17) $a_n = \left(\dfrac{n+2}{n+3}\right)^{n^2}$;

18) $a_n = \left(n \,\mathrm{sh} \dfrac{1}{n}\right)^{-n^3}$;

19) $a_n = \left(\dfrac{n-1}{n+1}\right)^{\sqrt{n^3+3n+1}}$;

20) $a_n = \dfrac{n^{n+1}}{(3n^2+2n+1)^{\frac{n+3}{2}}}$.

III. Să se cerceteze convergența seriei $\sum a_n$, folosind criteriul Raabe-Duhamel

1) $a_n = \dfrac{(2n-1)!}{(2n)!!}$;

2) $a_n = \dfrac{(n+1)!}{\beta(\beta+1)\cdot\ldots\cdot(\beta+n)n^{\alpha}}$, $\beta > 0$;

3) $a_n = \dfrac{n!\cdot e^n}{n^{n+\alpha}}$;

4) $a_n = \dfrac{\sqrt{n!}}{(a+\sqrt{2})(a+\sqrt{3})\cdot\ldots\cdot(a+\sqrt{n+1})}$, $a > 0$;

5) $a_n = \dfrac{1\cdot 4\cdot\ldots\cdot(3n-2)\cdot 2\cdot 5\cdot\ldots\cdot(3n+2)}{n!\cdot(n+1)\cdot 9^n}$.

IV. Să se cerceteze convergența seriei $\sum a_n$, folosind criteriul ”raportului”

(obținînd formula asimptotică $a_n \sim \dfrac{c}{n^{\alpha}}$, $n\to\infty$):

1) $a_n = \dfrac{1}{\sqrt{(3n+1)(3n+5)}}$;

2) $a_n = 1-\cos\dfrac{2\pi}{n}$;

3) $a_n = \ln\left(1+\dfrac{1}{n\sqrt[3]{n}}\right)$;

4) $a_n = \dfrac{3n+1}{(2n+1)^2}$;

5) $a_n = \dfrac{1}{\sqrt{n}}\operatorname{arctg}\dfrac{1}{2\sqrt{n}}$;

6) $a_n = \dfrac{1}{\sqrt[3]{n}}\arcsin\dfrac{1}{\sqrt[5]{n^4}}$;

7) $a_n = \dfrac{4n^2+5n^3\sqrt{n}+2}{3^{n^4}+1}$;

8) $a_n = \left(e^{\frac{1}{n}}-1\right)\sin\dfrac{1}{\sqrt{n+1}}$;

9) $a_n = \sin\dfrac{2n+1}{n^3+5n+3}$;

10) $a_n = n\operatorname{tg}\dfrac{n+2}{n^2+3}$;

11) $a_n = \dfrac{\ln\left(1+\sin\frac{1}{n}\right)}{n+\ln^2 n}$;

12) $a_n = \dfrac{\sqrt{n+1}-\sqrt{n-1}}{\sqrt{n+2}}$;

13) $a_n = \ln\left(\dfrac{1}{\cos\frac{2\pi}{n}}\right)$;

14) $a_n = \sqrt{\ln\left(1+\dfrac{2}{n}\right)\ln\dfrac{\sqrt{n}-1}{\sqrt{n}+1}}$, $n \geq 2$;

15) $a_n = \left(1-\dfrac{\ln n}{n}\right)^{2n}$.

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V. Să se cerceteze convergența seriei $\sum a_n$, folosind criteriul de comparație:

1) $a_n = \dfrac{5+2(-1)^{n+1}}{3^n}$;

2) $a_n = \dfrac{\operatorname{arctg} n}{n^2+1}$;

3) $a_n = \dfrac{\sin^2 3n}{n\sqrt{n}}$;

4) $a_n = \dfrac{\cos\dfrac{\pi}{4n}}{\sqrt[5]{2n^5-1}}$;

5) $a_n = \sin\dfrac{3+(-1)^n}{n^2}$;

6) $a_n = \dfrac{\ln n+\sin n}{n^2+2\ln n}$;

7) $a_n = \dfrac{\operatorname{arctg}\left(n^2+2n\right)}{n^2+2\ln 2}$;

8) $a_n = n^2 e^{-n}$;

9) $a_n = \dfrac{1}{n^2\ln n}$;

10) $a_n = \dfrac{n+2}{n^2\left(4+3\sin\dfrac{\pi n}{3}\right)}$;

11) $a_n = \dfrac{1}{n-\ln n}$;

12) $a_n = \dfrac{\ln(1+\ln n)}{\sqrt[4]{n^4+3n^2+1}\cdot\ln^3(n+2)}$.

VI. Să se cerceteze convergența seriilor $\sum a_n$, unde:

1) $\sum\limits_{n=1}^{\infty}\dfrac{1}{(2n-1)\cdot 2^{2n-1}}$;

2) $\sum\limits_{n=1}^{\infty}\sin\dfrac{\pi}{2^n}$;

3) $\sum\limits_{n=1}^{\infty}\dfrac{1+n}{1+n^2}$;

4) $\sum\limits_{n=1}^{\infty}\dfrac{1}{\ln(n+1)}$;

5) $\sum\limits_{n=1}^{\infty}\dfrac{1}{\sqrt{n^2+2n}}$;

6) $\sum\limits_{n=1}^{\infty}\dfrac{1}{n}\left(\sqrt{n+1}-\sqrt{n-1}\right)$;

7) $\sum\limits_{n=1}^{\infty} n\cdot\operatorname{tg}\dfrac{\pi}{2^{n+1}}$;

8) $\sum\limits_{n=1}^{\infty}\dfrac{2\cdot 5\cdot\ldots\cdot(3n-1)}{1\cdot 5\cdot\ldots\cdot(4n-3)}$;

9) $\sum\limits_{n=1}^{\infty}\dfrac{1}{\ln^n(n+1)}$;

10) $\sum\limits_{n=1}^{\infty}\dfrac{1}{3^n}\left(\dfrac{n+1}{n}\right)^{n^2}$;

11) $\sum\limits_{n=1}^{\infty}\dfrac{1}{\sqrt{n}}\ln\dfrac{2n+1}{2n-1}$;

12) $\sum\limits_{n=1}^{\infty}\dfrac{n!}{n^n}$;

13) $\sum\limits_{n=1}^{\infty}\dfrac{2}{5^{n-1}+n-1}$;

14) $\sum\limits_{n=1}^{\infty}\dfrac{1}{n-1}\operatorname{arctg}\dfrac{1}{\sqrt[3]{n-1}}$;

15) $\sum\limits_{n=1}^{\infty}\dfrac{n^3+2}{n^5+\sin 2^n}$;

16) $\sum\limits_{n=1}^{\infty}\dfrac{\left(n^2+3\right)^2}{n^5+\ln^4 n}$;

17) $\sum\limits_{n=1}^{\infty}\dfrac{1}{n-\cos^2 6n}$;

18) $\sum\limits_{n=1}^{\infty}\dfrac{1}{n^2-\ln n}$;

19) $\sum\limits_{n=1}^{\infty}\dfrac{1}{\sqrt{n+3}}\left(e^{\frac{1}{\sqrt{n}}}-1\right)$;

20) $\sum\limits_{n=1}^{\infty}\ln\dfrac{n^2+n+2}{n^2+1}$;

21) $\sum\limits_{n=1}^{\infty}\left(1-\cos\dfrac{\pi}{n}\right)$;

22) $\sum\limits_{n=1}^{\infty}\sin\dfrac{\sqrt[3]{n}}{\sqrt{n^5+2}}$;

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23) $\sum_{n=1}^{\infty}\frac{3+7n}{5^n+n}$;

24) $\sum_{n=1}^{\infty}\left(e^{\frac{\sqrt{n}}{n^2-1}}-1\right)$;

25) $\sum_{n=1}^{\infty}\frac{n^n}{(n!)^2}$;

26) $\sum_{n=1}^{\infty}n!\sin\frac{\pi}{2^n}$;

27) $\sum_{n=1}^{\infty}\left(\frac{n+2}{3n-1}\right)^n$;

28) $\sum_{n=1}^{\infty}\left(\frac{n+2}{3n-1}\right)^{n^2}$;

29) $a_n=\frac{1}{\sqrt[3]{n}}\sin\frac{\pi}{n^2}$;

30) $a_n=\frac{1}{n}\ln\left(1+\frac{1}{\sqrt{n+3}}\right)$;

31) $a_n=\frac{n\sin^2 2n}{\sqrt{n^7+4}}$;

32) $a_n=\frac{\operatorname{arctg}\sqrt{n+2}}{n\ln^3(n+2)}$;

33) $a_n=\frac{1}{n^5}\left(\frac{5n^2+7}{4n^2+10}\right)^{3n}$;

34) $a_n=\frac{n^n}{(n!)^2}$;

35) $a_n=n!\operatorname{arctg}\frac{2^n}{n^n}$;

36) $a_n=\frac{1}{\sqrt[n]{\ln n}}$;

37) $a_n=\frac{\sqrt{n+2}}{n+3}\ln\frac{3n-1}{3n+1}$;

38) $a_n=3^n\cos\frac{1}{n!}$;

39) $a_n=\frac{2^2\cdot 5^2\cdot 8^2\cdot\ldots\cdot(3n+2)^2}{(2n+1)!}\sin\frac{1}{3^n}$.

40) $\sum_{n=1}^{\infty}\frac{1}{(2n-1)\cdot 2^{2n-1}}$;

41) $\sum_{n=1}^{\infty}\sin\frac{\pi}{2^n}$;

42) $\sum_{n=1}^{\infty}\frac{1+n}{1+n^2}$;

43) $\sum_{n=1}^{\infty}\frac{1}{\ln(n+1)}$;

44) $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n^2+2n}}$;

45) $\sum_{n=1}^{\infty}\frac{1}{n}\left(\sqrt{n+1}-\sqrt{n-1}\right)$;

46) $\sum_{n=1}^{\infty}n\cdot\operatorname{tg}\frac{\pi}{2^{n+1}}$;

47) $\sum_{n=1}^{\infty}\frac{2\cdot 5\cdot\ldots\cdot(3n-1)}{1\cdot 5\cdot\ldots\cdot(4n-3)}$;

48) $\sum_{n=1}^{\infty}\frac{1}{\ln^n(n+1)}$;

49) $\sum_{n=1}^{\infty}\frac{1}{3^n}\left(\frac{n+1}{n}\right)^{n^2}$;

50) $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}\ln\frac{2n+1}{2n-1}$;

51) $\sum_{n=1}^{\infty}\frac{n!}{n^n}$;

52) $\sum_{n=1}^{\infty}\frac{2}{5^{n-1}+n-1}$;

53) $\sum_{n=1}^{\infty}\frac{1}{n-1}\operatorname{arctg}\frac{1}{\sqrt[3]{n-1}}$;

54) $\sum_{n=1}^{\infty}\frac{n^3+2}{n^5+\sin 2^n}$;

55) $\sum_{n=1}^{\infty}\frac{(n^2+3)^2}{n^5+\ln^4 n}$;

56) $\sum_{n=1}^{\infty}\frac{1}{n-\cos^2 6n}$;

57) $\sum_{n=1}^{\infty}\frac{1}{n^2-\ln n}$;

58) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+3}}\left(e^{\frac{1}{\sqrt{n}}}-1\right)$;

59) $\sum_{n=1}^{\infty} \ln \frac{n^2+n+2}{n^2+1}$;

60) $\sum_{n=1}^{\infty}\left(1-\cos\frac{\pi}{n}\right)$;

61) $\sum_{n=1}^{\infty} \sin\frac{\sqrt[3]{n}}{\sqrt{n^5+2}}$;

62) $\sum_{n=1}^{\infty} \frac{3+7n}{5^n+n}$;

63) $\sum_{n=1}^{\infty}\left(e^{\frac{\sqrt{n}}{n^2-1}}-1\right)$;

64) $\sum_{n=1}^{\infty} \frac{n^n}{(n!)^2}$;

65) $\sum_{n=1}^{\infty} n!\sin\frac{\pi}{2^n}$;

66) $\sum_{n=1}^{\infty}\left(\frac{n+2}{3n-1}\right)^n$;

67) $\sum_{n=1}^{\infty}\left(\frac{n+2}{3n-1}\right)^{n^2}$;