

I. Să se găsească domeniul de convergență a seriei funcționale:

1) $\sum_{n=1}^{\infty} 3^n x^n$;

6) $\sum_{n=1}^{\infty} \frac{n}{n^2+4} \left(\frac{x+2}{2x+1}\right)^n$;

2) $\sum_{n=1}^{\infty} \frac{1}{x^n}$;

7) $\sum_{n=1}^{\infty} \frac{1}{n(x+2)^n}$;

3) $\sum_{n=1}^{\infty} e^{-nx}$;

8) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \left(\frac{2x}{x^2+1}\right)^n$;

4) $\sum_{n=1}^{\infty} \frac{\ln^n x}{n}$;

9) $\sum_{n=1}^{\infty} \frac{x^n}{1-x^n}$;

5) $\sum_{n=1}^{\infty} \frac{\ln^n x}{n^2}$;

10) $\sum_{n=1}^{\infty} \frac{n!}{(x^2+1)(x^2+2)\cdots(x^2+n)}$.

II. Să se determine domeniul de convergență a seriei de puteri:

1) $\sum_{n=1}^{\infty} \frac{x^n}{n!}$;

10) $\sum_{n=1}^{\infty} \left(\frac{2n-1}{3n+2}\right)^n (x-2)^n$;

2) $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n!}$;

11) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n}} \left(\frac{x-1}{3}\right)^n$;

3) $\sum_{n=1}^{\infty} \frac{2^n}{\sqrt{n}} x^n$;

12) $\sum_{n=1}^{\infty} \frac{(x+1)^n}{\sqrt{n+1}} \ln \frac{3n-2}{3n+2}$; 13)

4) $\sum_{n=1}^{\infty} \frac{n^3+1}{4^n} x^n$;

$\sum_{n=1}^{\infty} \frac{(2n-1)!!}{n!} (x+2)^n$;

5) $\sum_{n=1}^{\infty} \frac{(-1)^n}{6^n \sqrt[3]{n}} x^n$

14) $\sum_{n=1}^{\infty} 4^{n^2} (x+1)^{n^2}$;

6) $\sum_{n=1}^{\infty} \frac{1}{(n+2)n!} x^n$

15) $\sum_{n=1}^{\infty} n! x^n$;

7) $\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n} x^n$

16) $\sum_{n=1}^{\infty} 3^{n^2} \cdot x^{n^2}$;

8) $\sum_{n=1}^{\infty} 25^n x^{2n}$;

17) $\sum_{n=1}^{\infty} \frac{n^n}{n!} x^n$;

9) $\sum_{n=1}^{\infty} \frac{(x+1)^n}{n\sqrt{n}}$;

18) $\sum_{n=1}^{\infty} \frac{(x-3)^{2n}}{n^2}$;

19)
$$\sum_{n=1}^{\infty} n^3(x+4)^n;$$

22)
$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{(x-1)^{2n}}{2n};$$

20)
$$\sum_{n=1}^{\infty} \frac{(x+2)^{n-1}}{n \cdot 7^n};$$

23)
$$\sum_{n=1}^{\infty} n^n(x+3)^n;$$

21)
$$\sum_{n=1}^{\infty} \frac{(x-3)^n}{n\sqrt{n+1}};$$

24)
$$\sum_{n=1}^{\infty} (-1)^n(3n+1)^2 x^n.$$

III. Folosind derivarea și integrarea termen cu termen, să se calculeze sumele următoarelor serii de puteri:

1)
$$\sum_{n=1}^{\infty} n \cdot x^{n-1};$$

6)
$$\sum_{n=1}^{\infty} \frac{n \cdot x^n}{2^n};$$

2)
$$\sum_{n=1}^{\infty} n \cdot x^n;$$

7)
$$\sum_{n=1}^{\infty} \frac{(x-3)^n}{n \cdot 2^n};$$

3)
$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n};$$

8)
$$\sum_{n=1}^{\infty} \frac{x^{2n}}{(2n-3)(2n-2)};$$

4)
$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{n+1}}{n(n+1)};$$

9)
$$\sum_{n=1}^{\infty} (3n^2 + 8n + 5)x^{n+2};$$

5)
$$\sum_{n=1}^{\infty} \frac{n(n+1)}{2} x^{n-1};$$

10)
$$\sum_{n=1}^{\infty} (n^2 - n + 1)x^n.$$