

# Logic function minimization



We can minimize the logic functions using the following minimization methods:

1. Analytical (covering, combines theorems)
2. Karnaugh maps (small number of variables)
3. Quine-McCluskey algorithm (can be performed for functions of an arbitrarily large number of variables using this tabular method that can be translated into a computer program).

# Karnaugh Maps

Each input combination with a 1 in the truth table corresponds to a minterm in the logic function's canonical sum. Since pairs of adjacent 1 cells in the Karnaugh map have minterms that differ in only one variable, the minterm pairs can be combined into a single product term.

We can simplify a logic function by combining pairs of adjacent 1-cells (minterms) whenever possible and write a sum of product terms that covers all of the 1-cells.

In general,  $2^i$  1-cells may be combined to form a product term containing  $n-i$  literals, where  $n$  is the number of variables in the function.



Graphically this rule means that we can circle rectangular sets of  $2^i$  1's. · If a circle covers only areas of the map where the variable is 0 then the variable is complemented in the product term.

- · If a circle covers only areas of the map where the variable is 1 – uncomplemented.
- · If a circle covers both areas of the map 0 and 1, then the variable does not appear in the product term.

Obs. The number of circles must be minimal and the number of 1's in each circle -maximal.

$$F = \sum (2,4,6,7,8,9,11,12,14,15)$$

	x1	x2	x3	x4	F
0	0	0	0	0	
1	0	0	0	1	
2	0	0	1	0	
3	0	0	1	1	
4	0	1	0	0	
5	0	1	0	1	
6	0	1	1	0	
7	0	1	1	1	
8	1	0	0	0	
9	1	0	0	1	
10	1	0	1	0	
11	1	0	1	1	
12	1	1	0	0	
13	1	1	0	1	
14	1	1	1	0	
15	1	1	1	1	

		x1x2			
		00	01	11	10
x3x4	00	0000	0100	1100	1000
	01	0001	0101	1101	1001
	11	0011	0111	1111	1011
	10	0010	0110	1110	1010

		x1x2			
		00	01	11	10
x3x4	00				
	01				
	11				
	10				

$$F = \sum (0,1,3,5,6,7,8,15)$$

	x1	x2	x3	x4	F
0	0	0	0	0	
1	0	0	0	1	
2	0	0	1	0	
3	0	0	1	1	
4	0	1	0	0	
5	0	1	0	1	
6	0	1	1	0	
7	0	1	1	1	
8	1	0	0	0	
9	1	0	0	1	
10	1	0	1	0	
11	1	0	1	1	
12	1	1	0	0	
13	1	1	0	1	
14	1	1	1	0	
15	1	1	1	1	

		x1x2			
		00	01	11	10
x3x4	00	0000	0100	1100	1000
	01	0001	0101	1101	1001
	11	0011	0111	1111	1011
	10	0010	0110	1110	1010

		x1x2			
		00	01	11	10
x3x4	00				
	01				
	11				
	10				

$$F = \sum (0,1,2,3,5,6,8,10,12)$$

	x1	x2	x3	x4	F
0	0	0	0	0	
1	0	0	0	1	
2	0	0	1	0	
3	0	0	1	1	
4	0	1	0	0	
5	0	1	0	1	
6	0	1	1	0	
7	0	1	1	1	
8	1	0	0	0	
9	1	0	0	1	
10	1	0	1	0	
11	1	0	1	1	
12	1	1	0	0	
13	1	1	0	1	
14	1	1	1	0	
15	1	1	1	1	

		x1x2			
x3x4		00	01	11	10
	00	0000	0100	1100	1000
01	0001	0101	1101	1001	
11	0011	0111	1111	1011	
10	0010	0110	1110	1010	

		x1x2			
x3x4		00	01	11	10
	00				
01					
11					
10					

$$F = \sum(1,2,3,4,5,6,9,10,11)$$

Minimal sum

x1x2 x3x4 \	00	01	11	10
00				
01				
11				
10				

Minimal product

x1x2 x3x4 \	00	01	11	10
00				
01				
11				
10				



$$F = \sum (0,2,4,5,6,8,10,12,14)$$

Minimal sum

		x1x2			
		00	01	11	10
x3x4	00				
	01				
	11				
	10				
	10				

Minimal product

		x1x2			
		00	01	11	10
x3x4	00				
	01				
	11				
	10				
	10				

# Minimisation of 5-variables logic functions

		x1x2x3							
		000	001	011	010	110	111	101	100
x4x5	00	0	4	12	8	24	28	20	16
	01	1	5	13	9	25	29	21	17
	11	3	7	15	11	27	31	23	19
	10	2	6	14	10	26	30	22	18

	x1	x2	x3	x4	x5	F
0	0	0	0	0	0	
1	0	0	0	0	1	
2	0	0	0	1	0	
3	0	0	0	1	1	
4	0	0	1	0	0	
5	0	0	1	0	1	
6	0	0	1	1	0	
7	0	0	1	1	1	
8	0	1	0	0	0	
9	0	1	0	0	1	
10	0	1	0	1	0	
11	0	1	0	1	1	
12	0	1	1	0	0	
13	0	1	1	0	1	
14	0	1	1	1	0	
15	0	1	1	1	1	
16	1	0	0	0	0	
17	1	0	0	0	1	
18	1	0	0	1	0	
19	1	0	0	1	1	
20	1	0	1	0	0	
21	1	0	1	0	1	
22	1	0	1	1	0	
23	1	0	1	1	1	
24	1	1	0	0	0	
25	1	1	0	0	1	
26	1	1	0	1	0	
27	1	1	0	1	1	
28	1	1	1	0	0	
29	1	1	1	0	1	
30	1	1	1	1	0	
31	1	1	1	1	1	

Alipiri corecte:

x1x2x3								
x4x5	000	001	011	010	110	111	101	100
00								
01								
11								
10								

Alipiri incorecte:

x1x2x3								
x4x5	000	001	011	010	110	111	101	100
00								
01								
11								
10								

$$F = \sum (1,4,5,6,8,9,14,15,17,20,21,22,24,26,29,30,31)$$

	x1	x2	x3	x4	x5	F
0	0	0	0	0	0	
1	0	0	0	0	1	
2	0	0	0	1	0	
3	0	0	0	1	1	
4	0	0	1	0	0	
5	0	0	1	0	1	
6	0	0	1	1	0	
7	0	0	1	1	1	
8	0	1	0	0	0	
9	0	1	0	0	1	
10	0	1	0	1	0	
11	0	1	0	1	1	
12	0	1	1	0	0	
13	0	1	1	0	1	
14	0	1	1	1	0	
15	0	1	1	1	1	
16	1	0	0	0	0	
17	1	0	0	0	1	
18	1	0	0	1	0	
19	1	0	0	1	1	
20	1	0	1	0	0	
21	1	0	1	0	1	
22	1	0	1	1	0	
23	1	0	1	1	1	
24	1	1	0	0	0	
25	1	1	0	0	1	
26	1	1	0	1	0	
27	1	1	0	1	1	
28	1	1	1	0	0	
29	1	1	1	0	1	
30	1	1	1	1	0	
31	1	1	1	1	1	

x1x2x3								
x4x5	000	001	011	010	110	111	101	100
00	0	4	12	8	24	28	20	16
01	1	5	13	9	25	29	21	17
11	3	7	15	11	27	31	23	19
10	2	6	14	10	26	30	22	18

x1x2x3								
x4x5	000	001	011	010	110	111	101	100
00								
01								
11								
10								



