

# Switching algebra



# Rules for BCD addition

1. If the sum is smaller or equal to 9 the addition is done without correction.
2. If after addition illegal combination appears or carry out occurs the correction is 6 (0110).
3. Carry out which appears after correction is added to the next tetrad.



# Switching Algebra

Switching algebra consist of :

- 1) A set of elements  $B = \{0,1\}$ ;
- 2) Logic operations AND, OR and NOT, that are defined as:

AND

$$0*0=0$$

$$0*1=0$$

$$1*0=0$$

$$1*1=1$$

OR

$$0+0=0$$

$$0+1=1$$

$$1+0=1$$

$$1+1=1$$

NOT

$$\overline{0} = 1$$

$$\overline{1} = 0$$

## Single-variable Theorems (Axioms)

1.  $X+0=X$      $X \cdot 1=X$                       - Identities

2.  $X+1=1$      $X \cdot 0=0$                       - Null element

3.  $X+X=X$      $X \cdot X=X$                       - Idem potency

4.  $(X')' = X$                       - Involution

5.  $X+X'=1$      $X \cdot X'=0$                       - Complements

| No | Logic expressions  |  | Theorem                |
|----|--|--|------------------------|
| 1. | $x_1 + x_2 = x_2 + x_1$  | $x_1 \cdot x_2 = x_2 \cdot x_1$                              | <i>Commutativity</i>   |
| 2. | $(x_1 + x_2) + x_3 = x_1 + (x_2 + x_3)$  | $(x_1 \cdot x_2) \cdot x_3 = x_1 \cdot (x_2 \cdot x_3)$      | <i>Associativity</i>   |
| 3. | $x_1 x_2 + x_1 x_3 = x_1 \cdot (x_2 + x_3)$  | $(x_1 + x_2) \cdot (x_1 + x_3) = x_1 + (x_2 \cdot x_3)$      | <i>Distributivity/</i> |
| 4. | $x_1 + x_1 x_2 = x_1$  | $x_1 \cdot (x_1 + x_2) = x_1$                                | <i>Covering</i>        |
|    | Proof: $x_1 + x_1 x_2 = x_1 \cdot 1 + x_1 x_2 = x_1(1 + x_2) = x_1 \cdot 1 = x_1$            |  |                        |
| 5. | $x_1 x_2 + x_1 \overline{x_2} = x_1$   | $(x_1 + x_2) \cdot (x_1 + \overline{x_2}) = x_1$             | <i>Combining</i>       |
|    | Proof: $x_1 x_2 + x_1 \overline{x_2} = x_1 \cdot (x_2 + \overline{x_2}) = x_1 \cdot 1 = x_1$ |  |                        |
| 6. | $\overline{x_1 \cdot x_2} = \overline{x_1} + \overline{x_2}$                                 | $\overline{x_1 + x_2} = \overline{x_1} \cdot \overline{x_2}$ | <i>De Morgan</i>       |



# Minimize logic expressions

$$x_1x_2 + x_1x_2\bar{x}_3x_4 + x_1x_2x_4\bar{x}_5 + x_1x_2\bar{x}_3x_5 + \bar{x}_3x_4x_5$$

$$x_1x_2x_3x_4 \cdot (x_1x_2x_3\bar{x}_4 + x_1\bar{x}_2x_3x_4 + \bar{x}_1x_2x_3x_4 + x_1x_2\bar{x}_3x_4)$$

$$x_1x_2x_3 + x_3(\overline{x_1\bar{x}_2} \cdot \overline{\bar{x}_1x_3})$$

# Minimize logic expressions

$$(\bar{x}_1 + x_2)(x_3 \cdot (\bar{x}_4 + x_5)) + (\bar{x}_1 + x_2)\overline{(x_1 \cdot (\bar{x}_2 + x_3))}$$

$$x_1 x_2 x_3 + x_2 \overline{\overline{(x_1 \bar{x}_2 x_3 \cdot \bar{x}_1 x_3)}} + \overline{\bar{x}_1 + \bar{x}_2 + \bar{x}_3 + x_4}$$

# Write Truth tables

$$F = \bar{x}_1x_2 + \bar{x}_1\bar{x}_2x_3$$

|   | x1 | x2 | x3 | F |
|---|----|----|----|---|
| 0 |    |    |    |   |
| 1 |    |    |    |   |
| 2 |    |    |    |   |
| 3 |    |    |    |   |
| 4 |    |    |    |   |
| 5 |    |    |    |   |
| 6 |    |    |    |   |
| 7 |    |    |    |   |



# Write Truth tables

$$F = x_1 + \bar{x}_2(\bar{x}_3 + x_4)$$

|   | x1 | x2 | x3 | F |
|---|----|----|----|---|
| 0 |    |    |    |   |
| 1 |    |    |    |   |
| 2 |    |    |    |   |
| 3 |    |    |    |   |
| 4 |    |    |    |   |
| 5 |    |    |    |   |
| 6 |    |    |    |   |
| 7 |    |    |    |   |



# Write Truth tables

$$F = \overline{x_1 + x_2 + x_3 + x_4}$$

|   | x1 | x2 | x3 | F |
|---|----|----|----|---|
| 0 |    |    |    |   |
| 1 |    |    |    |   |
| 2 |    |    |    |   |
| 3 |    |    |    |   |
| 4 |    |    |    |   |
| 5 |    |    |    |   |
| 6 |    |    |    |   |
| 7 |    |    |    |   |

Write canonical sum and  
canonical product

$$F = \sum (2,4,6,7)$$

$$F = \prod (0,1,7)$$

$$F = x_1 + \bar{x}_2\bar{x}_3$$